

(2)



RADC-TR-80-78, Vol I (of two) Final Technical Report April 1980

ACOSS FOUR (ACTIVE CONTROL OF SPACE STRUCTURES) THEORY

The Charles Stark Draper Laboratory, Inc.

Sponsored by Defense Advanced Research Projects Agency (DoD) ARPA Order No. 3654



APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Defense Advanced Research Projects Agency or the U.S. Government

ROME AIR DEVELOPMENT CENTER Air Force Systems Command Griffiss Air Force Base, New York 13441

INC FILE COP!

This report has been reviewed by the RADC Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public, including foreign nations.

RADC-TR-80-78, Vol I (of two) has been reviewed and is approved for publication.

Richard W. Carman

APPROVED:

RICHARD W. CARMAN Project Engineer

If your address has changed or if you wish to be removed from the RADC mailing list, or if the addressee is no longer employed by your organization, please notify RADC (OCSE), Griffiss AFB NY 13441. This will assist us in maintaining a current mailing list.

Do not return this copy. Retain or destroy.

ACOSS FOUR (ACTIVE CONTROL OF SPACE STRUCTURES) THEORY

R.R. Strunce

D.R. Hegg

J.G. Lin

T.C. Henderson

Contractor: The Charles Stark Draper Laboratory. Inc.

Contract Number: F30602-78-C-0268

Effective Date of Contract: 4 July 1978 Contract Expiration Date: 31 December 1979 Short Title of Work: Active Structures

Program Code Number: 8E20

Period of Work Covered: 24 Jul 78 - 31 Dec 79

Principal Investigator: Robert R. Strunce
Phone: 617-258-1547

Project Engineer:

Richard W. Carman

Phone: 315-330-3148

Approved for public release; distribution unlimited.

This research was supported by the Defense Advanced Research Projects Agency of the Department of Defense and was monitored by Richard W. Carman (OCSE), Griffiss AFB NY 13441 under Contract F30602-78-C-0268

	SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)	
(0)	REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
(lo)	,	3. RECIPIENT'S CATALOG NUMBER
	RADC/TR-80-78, Vol I (of two) / AD-A085654	
	4. TITLE (and Subtitle)	5. TYPE OF REPORT & PENSON COMERED.
	LACUSS BUILD CAULIUM CUNTRUL UN SPACE / / 🔻 🛩	Final Aechnical Report
	STRUCTURES) THEORY TALLING I.	24 July 78—31 Dec 79
\sim L		R-1338 (
(10)	T. MUTHORINA TIQUAN	S. CONTRACT OR GRANT NUMBER(s)
Robert	R./Strunce G./Lin \ (/5/	F39602-78-C-9268
Daniel	R. Hegg TACC. Henderson	
L Voi	limothy /	ARPA Order-3654
	The Charles Stark Draper Laboratory, Inc.	AREA & WORK UNIT NUMBERS
	Cambridge MA 02139	62301E (17)0/(
		C6540104
	11. CONTROLLING OFFICE NAME AND ADDRESS	12REPORT SATE
	Defense Advanced Research Projects Agency	Apr 1 1080
	1400 Wilson Blvd	13. NUMBER OF PAGES
	Arlington VA 22209 14. MONITORING AGENCY NAME & ADDRESS(II dillerent from Controlling Office)	15. SECURITY CLASS. (of this report)
	Rome Air Development Center (OCSE)	UNCLASSIFIED
	Griffiss AFB NY 13441	
	(12)/67/	15a. DECLASSIFICATION/DOWNGRADING
	16. DISTRIBUTION STATEMENT (of this Report)	N/A SCREET
i	To story	J
	Approved for public release; distribution unli	mited
,		
	17. DISTRIBUTION STATEMENT (of the abetract entered in Block 20, if different from	m Report)
	Same	i
	18. SUPPLEMENTARY NOTES	
	RADC Project Engineer: Richard W. Carman (OCS	. ₂ \
	RADO Ploject Engineer: Richard W. Carman (OCS	' ^E '
	19. KEY WORDS (Continue on reverse side if necessary and identify by block number)	
		Damping Augmentation
	Structural Vibration Control Control Sp	
	Tetrahedral Model Observatio	n Spillover
	20. ABSTRACT (Continue on reverse side if necessary and identity by block number) This is the Charles Stark Draper Laboratory, I	nc. final necknical
	report on its Actively Controlled Structures T	
	objective of the research reported here was to	
	and analytical tools to support the successful	
	vibration control of large flexible spacecraft	
Í	theory and applications were initiated. For t	
	several representative design methods were sel	

DD 1 JAN 73 1473 EDITION OF 1 NOV 68 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

428386

LB

UNCLASSIFIED

ENCURITY CLASSIFICATION OF THIS PAGE(When Date Entered)

focusing on an examination of the theoretical basis for each method and potential difficulties associated with their use in reduced-order large space structure controller design. The methods initially selected are characterized by constant-gain output feedback, the simplest form of active multivariable control; (1) Modal Decoupling, (2) Pole Assignment,

- (3) Optimal Output Feedback, (4) Suboptimal Output Feedback,
- (5) Stochastic Optimal Output Feedback. A performance comparison of specific designs with these methods was made. Extensions to the published Kosut methods of suboptimal output feedback are developed, as well as the details of an algorithm necessary for a numerical solution. Techniques and conditions are developed for reduction of control (observation) spillover by placement of actuators (sensors), by synthesis of the actuator (sensor) influences, and by compensation of actuators (sensors). For the applications effort, relatively high order models representative of the large space structures of interest were employed. Effectiveness of both passive and active local member dampers as well as modern modal controller feedback designs for inducing vibration damping, was studied by simulation. A simple structural model (tetrahedron) was developed for the purpose of evaluating various large space structure control methods.

ACKNOWLEDGMENT

The work reported herein was performed by The Charles Stark Draper Laboratory, Inc. (CSDL) under Contract F30602-78-C-0268. The research was supported by the Advanced Research Projects Agency of the Department of Defense, and was monitored by the Rome Air Development Center. This final report covers the time period from 24 July 1978 to 31 December 1979. The technical monitors of this program are Major A. Herzberg and Mr. R. Carman.

The Program Manager is Dr. Keto Soosaar and the Principal Investigator is Mr. Robert Strunce. This study was performed within the Advanced Systems Department headed by Mr. David Hoag. The contributors to this report are Dr. Jiguan G. Lin (Section 2), Dr. Daniel R. Hegg (Sections 1, 3), Mr. Robert R. Strunce (Sections 1, 4), and Mr. Timothy C. Henderson (Appendix A).

1		
Acces	sion For	
	ILLASO	
DDC T		
	ounced	
Justin	fication	
Ву		
Distri	butien/	
Avei	ability C	edes
	Avaid and	or/
Dist	special	ł
A		

TABLE OF CONTENTS

Volume 1

Section			Page
1	EXECU	TIVE SUMMARY	1
	1.1	Goals and Approaches	1
	1.2	Research Scope	
	1.3	Summary of Principal Results	3
	1.4	Conclusions and Recommendations	6
2		TION OF CONTROL AND OBSERVATION SPILLOVER IN TION CONTROL OF LARGE FLEXIBLE SPACE STRUCTURES	8
	2.1	Introduction	8
	2.2	Placement of Actuators	. 12
	2.3	Synthesis of Actuator Influences	. 17
	2.4	Compensation of Actuator Inputs	. 25
	2.5	Three-Step Combined Approach	. 29
	2.6	Conclusions	. 31
3	OUTPU	T FEEDBACK	. 33
	3.1	Synopsis of Previous Research	. 33
	3.2	Scope of Current Research	. 34
	3.3	Preliminaries From Analysis	. 36
	3.4	Preliminaries From Numerical Analysis	. 40
	3.5	Algorithm for Solving Kosut's Gain Equation	. 42
	3.6	Status of Current Research	. 46
	3.7	Appendices	. 46
4		TURAL DAMPING AUGMENTATION AND MODERN MODAL COLLER DESIGN METHODOLOGY	. 52
	4.1	Introduction	. 52
	4.2	Tetrahedral Model	. 52
	4.3	Structural Damping Augmentation	. 53
	4.4	Modern Modal Controller	. 55
	4.5	Structural Damping Augmentation and Modern Modal Controller	. 57
	4.6	Summary and Conclusions	

SECTION 1

EXECUTIVE SUMMARY

1.1 Goals and Approaches

This report is submitted in fulfillment of the final documentation requirements for the study: Actively Controlled Structures Theory. Stringent attitude control requirements have been placed upon certain types of future large flexible space structures. In the light of these requirements and the inherent flexibility of these structures, it becomes essential to consider active control of structural vibrations to a degree not encountered in current spacecraft[1]* as a necessary component of an overall strategy for successful attitude control. The objective of the research reported here was to develop the theoretical and analytical tools to support the successful implementation of active vibration control for large flexible spacecraft.

The approaches to this objective assume the validity of finite element representations of sufficiently high order as a model for the class of structures of interest. This permits viewing the plant to be controlled as a finite-dimensional multivariable system; studies of control concepts and controller design techniques for such systems have been appearing in the literature for some time. 12,3] However, the order of a valid finite-element model is generally too high to use for control design by known methods; a reducedorder model must be used. This forces a reexamination of the potential applicability of existing multivariable controller design methods to large space structures. In the research program reported on here, parallel efforts in theory and applications were initiated. For the theoretical effort, several representative design methods were selected for careful study focusing on an examination of the theoretical basis for each method and a search for any potential difficulties associated with their use in reduced-order Large Space Structures (LSS) controller design. [4] The methods initially selected are characterized by constant-gain output feedback, the simplest form of active multivariable control. Considerable attention was given to developing new results, including modifications to the selected methods that may improve their suitability for LSS controller design. Performance comparisons between controller designs using the various methods applied to a common low-order structural model were made. Several of the methods that appeared most promising were selected for additional development and evaluation using higher order structural models. For the applications effort, relatively high order models representative of the large space structures of interest were employed. Effectiveness of both passive and active local member dampers, as well as modern modal controller feedback designs for inducing vibration damping, was studied by simulation. The active damping techniques were studied both separately and in combination. The effects of actuator dynamics upon member damper effectiveness were also studied.

Bracketed superscript numerals refer to similarly numbered items in the List of References at the end of the section.

The contents of this report are briefly summarized in the remainder of Section 1: the scope of our research is stated, a summary of the principal results obtained is given, and overall conclusions and recommendations based on all of the work performed under this contract are given.

Controller designs using reduced-order finite-element structural models lead to the phenomenon of control and observation spillover^[6] when the controller is used with a higher order structural model. Section 2 reports new results and new techniques for reduction of spillover. Conditions for elimination of spillover to selected residual modes and conditions for reduction of spillover to other residual modes are given. A three-step approach is proposed. The theory is demonstrated by a numerical example on the tetrahedral model (cf Appendix A).

Extensions to the Kosut method of Suboptimal Output Feedback relevant to LSS applications were reported previously in Chapter 6 of Reference 4. In order to implement these extensions, the delicate problem of numerical solution of rank-deficient linear algebraic systems must be solved. Progress in developing an algorithm to treat this question is reported in Section 3. A numerical example comparing calculations using the algorithm with calculations performed previously [4] for a two-mode model is given.

Design applications using modern modal control methodology and augmented by member dampers was presented in Reference 5. Section 4 illustrates the application of these techniques to the tetrahedral model.

A finite-element model fairly representative of realistic large space structures of interest, but of reasonably low order, and having the shape of a tetrahedron, has been developed as a common vehicle on which to evaluate various active control techniques. A complete description of this model is given in Appendix A.

1.2 Research Scope

1.2.1 Scope of Theoretical Research

Successful active control of large flexible space structures requires mastery of at least the following principal topics:

- (1) Development of high-fidelity structural models, including expected disturbances.
- (2) Reduction to lower-order models suitable for the design of feedback controllers.
- (3) Design of an active controller.
- (4) Stability analysis—including robustness.
- (5) Performance verification.

The final two topics include evaluation relative to a high-order structural model.

Although each of these topics has been considered in some degree, the theoretical efforts have concentrated on the controller design process—in particular, finite-dimensional control of linear multivariable systems. In addition to examining existing design methods, specific attention has been devoted to alleviating the adverse effects of control and observation spillover associated with reduced-order models, and to developing algorithms for numerical evaluation of new design techniques. To date, studies of methods involving dynamic compensation have been postponed, since the attendant complexity in both design and implementation of such methods is particularly unattractive for large-scale systems if a feasible alternative in the class of constant-gain output feedback methods can be found. For analysis of flight vehicles, structural models need to be sufficiently accurate so as to reflect the dynamics of sensors and actuators, as well as nonlinear dynamic characteristics of the entire structure of interest. Stability and performance estimates and tradeoffs based on less representative models are certainly open to serious question. Nevertheless, attempts to gain insight into reduced-order multivariable controller design appropriate to large space structures can be impeded by working with unnecessarily complex structural models at the concept development stage. Since the controller design problem is far from solved even for linear structural models which assume ideal sensors and actuators, the theoretical studies have been carried out in this context.

1.2.2 Scope of Applications Research

Studies were conducted to identify the problems encountered during the process of designing active vibration controllers. Two structures which best represent LSS structural characteristics were chosen; a realistic LSS structure, [5] and the tetrahedral structure (cf Appendix A). A nominal and a perturbed finite-element model of the realistic LSS structure were developed. During the control law design evaluation, a maximum of 50 modes and 32 colocated sensor/actuator locations were retained. The nominal and perturbed finite-element models for the tetrahedral structure had a maximum of 12 modes and 6 colocated sensor/actuator locations. The nominal finite-element models were employed in the controller design process and performance evaluations. Control law sensitivity to structural parameter changes was determined by evaluating the designs with respect to the perturbed model. Various sensor and actuator types (e.g., member dampers, force effectors, torque effectors) were utilized in order to establish preliminary instrument specifications. Actuator models were introduced for the purpose of assessing the impact of actuator dynamics upon overall closed-loop system stability and performance. The active modal controller design methods investigated were the Canavin local-damper concept and the Balas modern modal controller approach. These methods were chosen because they represent the extremes in terms of sophistication, stability and performance.

1.3 Summary of Principal Results

1.3.1 Results from Theoretical Research

In-depth theoretical analysis and preliminary performance comparison on the following five design methods were reported in Reference 4.

- (1) Modal Decoupling (Canavin).
- (2) Pole Assignment (Davison-Wang).
- (3) Optimal Output Feedback (Levine-Athans).
- (4) Suboptimal Output Feedback (Kosut).
- (5) Stochastic Optimal Output Feedback (Johnson).

A synopsis of that research is given in Section 3.1 of the present report. The principal results are briefly summarized. New and more precise stability theorems related to the Canavin method were proved. A coherent synthesis of the voluminous literature on the Davison-Wang method was given, including identification of several areas where significant improvements are possible. Extensions of the Kosut method which enable its application to systems with redundant sensor configurations were made. This gives the potential for significant improvements in performance. The Johnson method, still in development, demonstrates how coupling between critical and residual modes can be enforced so as to enhance stability. This is an original concept which contrasts sharply with the usually adverse effects upon stability and performance associated with control and observation spillover. Study of the Levine-Athans method produced an essentially negative result: the computational difficulties associated with its application seem to outweigh its advantages. These are simply the highlights of results from the study of design methods; a complete statement of the results and their significance is given in Reference 4.

Theoretical results documented in the present report concern new techniques for reduction of control and observation spillover (Section 2), and a proposed algorithm for the numerical implementation of the extended Kosut method (Section 3). In Section 2, a systematic three-step approach for reducing control spillover is given. The steps are: placement of actuators, where possible; synthesis of actuator influences, given actuator placement constraints; and compensation of actuator inputs to attenuate the action of modes which contribute to control spillover. Decoupling of influential residual modes from the closed-loop system can be achieved via proper synthesis of actuator (or sensor) influences. Conditions are given which make precise the nature and extent of control spillover reduction possible. The techniques and conditions are extendable to the dual case of observation spillover. Section 3 delineates a careful development of an algorithm for implementing the Kosut design method. Some sophisticated mathematical techniques are required because of the algebraic degeneracy associated with the extensions to this method reported in Reference 4. The important concepts of the generalized inverse of, and the singular value decomposition of, an arbitrary rectangular matrix are condensed from the diverse literature on the subject into a concise but complete exposition. This forms the basis for developing the algorithm. Pitfalls in attempting more obvious "brute force" approaches to an algorithm are exposed.

Research on the numerical implementation of the extended Kosut method is continuing. It is expected that simulations will verify that the implementation question at the design model level is solved by the algorithm proposed in Section 3. The next major task is to develop a systematic technique for using free parameters appearing in the controller design process to achieve the potential for performance improvement possible at the evaluation model level. A later task of much significance for the Kosut method is to characterize those classes of systems for which a stable closed-loop design using the Kosut method is (or is not) possible.

1.3.2 Results from Applications Research

Maximum sensed velocity

Nominal force/torque output

For a particular slew maneuver of a realistic LSS structure, a nominal set of damping requirements was established, which lead to the preliminary sensor/actuator requirements shown in Table 1-1 [see Section 2 of Reference 5].

Member Dampers	Force Effectors	Torque Effectors 100 rad/s 24 µrad	
100 rad/s	100 rad/s		
4 μm	500 μm		
	100 rad/s	Member Dampers Effectors 100 rad/s 100 rad/s	

150 um/s

75 N

8000 µm/s

4000 N

600 µrad/s

300 N-m

Table 1-1. Preliminary sensor/actuator requirements for vibration control.

The local-member-damper concept was employed to provide structural damping augmentation as described in Section 4 of Reference 5. Although this approach was insensitive to large parameter variations, the amount of damping per individual mode was unpredictable, as well as the fact that certain modes experienced a limit in achievable modal damping. This method provides a viable approach for obtaining low levels of damping over a broad frequency spectrum. Theoretical results show that this method is always Liapunov stable; this conclusion is no longer valid when sensor/actuator dynamics are introduced. Instability occurs when the phase lag of the sensor/actuator exceeds 90 degrees within the controlled frequency bandwidth.

The modern modal controller designs described in Section 5 of Reference 5 yield optimistic results when analyzed on reduced-order evaluation models (9 modes). When higher order evaluation models (50 modes) were used, observation and control spillover resulted in overall closed-loop instability. In addition, system performance degraded as a function of small changes in inherent structural damping.

In order to offset the adverse effects of observation and control spillover, structural damping augmentation controller designs (local-member-damper concept) were incorporated into the modern modal controller design process as described in Section 4 of the present report. Overall closed-loop system stability was maintained even when evaluated with high-order models.

1.4 Conclusions and Recommendations

For successful control of large space structures, it is recognized by the LSS community at large that the fundamental problem is the design of a finite-dimensional compensator to control an infinite dimensional system. To date, numerous theoretical contributions towards "solving" this problem have been made; however, the resolution of this problem requires further theoretical research which must be validated through appropriate design, analysis and experimentation. As a result of our endeavors, together with exposure to a broad spectrum of knowledge provided by the LSS community, it is our judgment that the technical issues in LSS control technology include the following.

- LSS Modeling accuracy should be known to within some specified bounds. Modeling errors will limit achievable control system performance. These errors may be introduced through initially assumed structural properties or the truncation process implicit in the finite-element method. In space, LSS parameters may vary as a function of thermal gradients, configuration changes, or depletion of consumables. The more strignent the mission performance requirements, the greater the LSS model fidelity required.
- (2) Upper Atmospheric Models must be improved and verified by appropriate experiments. Accurate knowledge of the external forces (e.g., earth magnetic and gravitational fields, solar wind and radiation pressure, drag) acting upon a LSS is necessary to satisfy precision control requirements.
- (3) System Identification is necessary for the purpose of LSS structural model verification. Parameter identification techniques must be developed such that modal frequencies, damping ratios, and mode shapes can be accurately determined. Consideration must be given to the type of sensors, onboard processing requirements, data reduction, and post-processing requirements.
- (4) Sensor and Actuator specifications must be determined in order to assess the applicability of existing hardware, as well as to provide new directions in research and development.
- (5) Control Law Design Methodology must address the following:
 - (a) The model reduction process which reduces the highdimensional finite-element model to a lower order design model.
 - (b) The design method for determining reduced-order compensators.

- (c) The criteria for determining overall closed-loop system stability.
- (d) Direct digital design methodologies and implementation techniques.
- (e) Sensor and actuator placement techniques which yield maximum observability and controllability.
- (f) Sensor and actuator dynamics.
- (g) The dynamic interaction of the attitude, vibration and, shape control laws.

LIST OF REFERENCES

- 1. Croopnick, S.R., Y.H. Lin and R.R. Strunce, "A Survey of Automatic Control Techniques for Large Space Structures," Proc. 8th IFAC Symposium on Automatic Control in Space, Oxford, England, July 2-6, 1979.
- 2. Gilbert, E.G., "Controllability and Observability in Multivariable Control Systems", J. SIAM Control, Vol. 2, No. 1, pp. 128-151, 1963.
- 3. Rosenbrock, H.H., State-space and Multivariable Theory. Wiley: New York, 1970.
- 4. Lin, J.G., Y.H. Lin, D.R. Hegg, T.L. Johnson, and J.E. Keat, Theory of Design Methods, Interim Technical Report, Actively Controlled Structures Theory, R-1249, Volume 1, The Charles Stark Draper Laboratory, Inc., April 1979.
- 5. Strunce, R., and T. Henderson, Application of Design Methods, Interim Technical Report, Actively Controlled Structures Theory, R-1249, Volume 2, The Charles Stark Draper Laboratory, Inc., April 1979.
- 6. Balas, M.J., "Active Control of Flexible Systems", J. Optimization Theory Appl., Vol. 25, No. 3, pp. 415-436, July 1978.

SECTION 2

REDUCTION OF CONTROL AND OBSERVATION SPILLOVER IN VIBRATION CONTROL OF LARGE FLEXIBLE SPACE STRUCTURES

2.1 Introduction

"Control spillover" and "observation spillover" surface as major road blocks to the application of state-of-the art control techniques to vibration control of large space structures. This section presents several approaches to their elimination or reduction.

Large space structures (LSS) deployable or errectable by the Space Shuttle are flexible, lightweight, and lightly damped. This new class of sætellites is characterized by increased susceptibility to natural and onboard vibration disturbances; active controllers are required for efficient vibration suppression. Complex flexible structures commonly are analyzed by the method of finite elements; finite-dimensional linear lumped-parameter models are generated (e.g., by computer program NASTRAN). Various state-ofthe-art control techniques (e.g., linear-quadratic regulation, pole placement) are used for designing LSS vibration controllers. Finite-element models for LSS are of very high order, whereas controller design using these techniques is possible only for systems of very low order because of computational requirements and computer capability. The finite-element models are therefore truncated with respect to modes of vibration, and the control of only a small number of modes is considered. As a result of modal truncation, control spillover and observation spillover interfere with the performance of vibration controllers thus designed. Balas $^{[1]*}$ has demonstrated that even for a simple flexible beam, control and observation spillover can cause closed-loop instability of an open-loop stable beam.

To be specific, consider the following standard finite-element modal representation of LSS

$$\ddot{\eta} + 2Z\Omega\dot{\eta} + \Omega^2 \eta = \Phi^T B_{p} u \qquad (2-1)$$

$$y = C_D \phi \eta + C_V \phi \dot{\eta}$$
 (2-2)

where Z = diag{ ζ_1 , ..., ζ_L }, Ω = diag{ ω_1 , ..., ω_L }, and Φ = [ϕ_1 , ..., ϕ_L] are L × L matrices; ζ_1 , ω_1 , ϕ_1 , denote respectively the natural damping ratio, frequency, and shape of the ith normal mode of vibration. (Natural damping in LSS is negligibly small; ζ_1 is currently considered to be about 0.005.)

^{*} Bracketed superscript numerals refer to similarly numbered items in the List of References at the end of the section.

 $\eta = (\eta_1, \, \dots, \, \eta_L)$ denotes the corresponding normal coordinates. $u = (u_1, \, \dots, \, u_n)$ denotes the control inputs to m force actuators and B_F the L × m influence matrix. $y = (y_1, \, \dots, \, y_\ell)$ denotes the observation outputs from ℓ_D displacement sensors and ℓ_V velocity sensors (0 $\leq \ell_D \leq \ell$, 0 $\leq \ell_V \leq \ell$); C_D and C_V denote the corresponding ℓ × L influence matrices. Superscript T denotes transpose.

In general, a very large number of vibration modes are included in model (2-1)-(2-2). Let $\{\omega_{Pi}, \phi_{Pi}\}$, $i=1,\ldots,N$, denote a selected subset for suppression; we call them primary modes for a self-explanatory reason. Completely neglecting all nonprimary modes yields the following truncated form of model (2-1)-(2-2).

$$\ddot{\eta}_{\mathbf{p}} + 2Z_{\mathbf{p}}\Omega_{\mathbf{p}}\dot{\eta}_{\mathbf{p}} + \Omega_{\mathbf{p}}^{2}\eta_{\mathbf{p}} = \Phi_{\mathbf{p}}^{\mathbf{T}}B_{\mathbf{p}}u \qquad (2-3)$$

$$y = C_{\mathbf{p}} \Phi_{\mathbf{p}} \Pi_{\mathbf{p}} + C_{\mathbf{v}} \Phi_{\mathbf{p}} \mathring{\Pi}_{\mathbf{p}}$$
 (2-4)

However, vibration controllers thus designed may not actually perform as desired; the neglected modes may significantly alter the desired performance and even destabilize the closed-loop systems. Let $\{\omega_{Rj},\phi_{Rj}\}$, j=1, ..., L-N, denote the residual, nonprimary modes. Then the finite-element model (2-1)-(2-2) can be partitioned into two parts as follows.

$$\ddot{\eta}_{\mathbf{p}} + 2Z_{\mathbf{p}}\Omega\dot{\eta}_{\mathbf{p}} + \Omega_{\mathbf{p}}^{2}\eta_{\mathbf{p}} = \Phi_{\mathbf{p}}^{\mathsf{T}}B_{\mathbf{p}}\mathbf{u}$$
 (2-5)

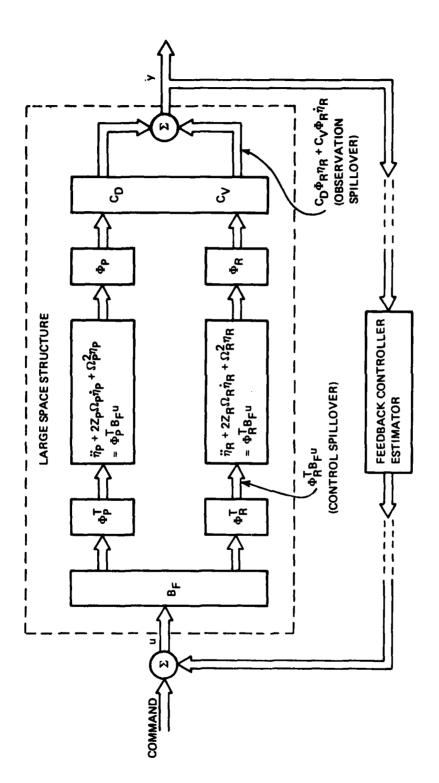
$$\ddot{\eta}_R + 2Z_R \Omega_R \dot{\eta}_R + \Omega_R^2 \eta_R = \Phi_R^T B_F u \qquad (2-6)$$

$$y = (C_{D}^{\phi}_{P}^{\eta}_{P} + C_{V}^{\phi}_{P}^{\dot{\eta}}_{P}) + (C_{D}^{\phi}_{R}^{\eta}_{R} + C_{V}^{\phi}_{R}^{\dot{\eta}}_{R})$$
 (2-7)

with Z_R , Ω_R , and Φ_R similarly defined, and $\eta=(\eta_P,\eta_R)$. Figure 2-1 illustrates that input u to control the primary part may "spill" over to the residual part; Balas [1] called the term $\Phi_R^T B_F u$ in (2-6) control spillover. It also illustrates that output y to observe the primary part may be "spilled" over by motions in

^{*} All forces, displacements, and velocities are in the generalized sense.

^{**} See Section 2.5 for discussions on their selection. The number N must be small enough to make the design and implementation of the desired control system computationally feasible within constraints.



. .. .

Figure 2-1. Control spillover, observation spillover.

the residual part; Balas [1] called the terms $C_D \Phi_R n_R$ and $C_V \Phi_R \hat{n}_R$ observation spillover. It was demonstrated in Reference 1 that the presence of both control and observation spillover could even destabilize a simple flexible beam. Control spillover without observation spillover can increase response time. [2] As a matter of fact, modeling errors such as observation spillover have long been recognized as the cause of divergence in Kalman filters. [3,4]

Simultaneous existence of control and observation spillover need not always be disastrous; it may be properly utilized to improve closed-loop stability as shown by examples in Reference 5. However, no general procedure for such an exploitation has yet been developed, and it is expected to be rather complicated since sufficient information on all the (infinitely many) modes of vibration is required. In this section, we consider only their reduction.

For eliminating observation spillover, Balas [1] suggested that the sensor data be prefiltered with a comb filter. For reducing control spillover, Sesak[6] augmented the regular quadratic performance index by a quadratic penalty on control spillover to some remaining modes; this results in a modification on the weighting matrix on control inputs. The reduction of observation spillover is similarly handled in the dual manner. [6]

The reduction of control spillover to, and observation spillover from, nonprimary modes $\{\omega_{Rj},\phi_{Rj}\}$ in the following absolute sense is discussed in this report.

$$\begin{split} & \phi_{Rj}^T B_F u(t) \approx 0 \qquad \text{for all values of } u(t). \\ & C_D \phi_{Rj} \eta_{Rj}(t) \approx 0 \qquad \text{for all values of } \eta_R(t). \\ & C_V \phi_{Rj} \eta_{Rj}(t) \approx 0 \qquad \text{for all values of } \eta_R(t). \end{split}$$

In other words, "elimination" will specifically mean

$$\begin{cases} \phi_{Rj}^T B_F = 0 \\ c_D \phi_{Rj} = 0 \\ c_V \phi_{Rj} = 0 \end{cases}$$
 (2-8)

and, "reduction" will mean

$$\begin{cases} \phi_{\mathbf{R}\mathbf{j}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} & \approx & 0 \\ \mathbf{C}_{\mathbf{D}} \phi_{\mathbf{R}\mathbf{j}} & \approx & 0 \\ \mathbf{C}_{\mathbf{V}} \phi_{\mathbf{R}\mathbf{j}} & \approx & 0 \end{cases}$$

$$(2-9)$$

A three-step approach to systematic spillover reduction is proposed here.

- Step 1 Elimination by proper placement of actuators and sensors.
- Step 2 Elimination by proper synthesis of actuator and sensor influences.
- <u>Step 3</u> Reduction by proper compensation of actuator inputs and sensor outputs.

Sections 2.2 through 2.4 discuss each step as an independent approach. The three different approaches are then combined in Section 2.5. Because of the duality between control and observation, we concentrate on the reduction of control spillover. Discussions and results on control spillover reduction can easily be extended to observation spillover reduction by the duality arguments.

2.2 Placement of Actuators

Undoubtedly, reduction of control spillover should originate from proper placement of the actuators on the structure to be controlled. In this section we shall first discuss the complete elimination of control spillover (an idealistic solution), and then the elimination of control spillover to selected nonprimary modes (a realistic solution).

The following simple theorem offers an ideal way of placing the actuators for complete elimination of control spillover.

Theorem 1:

If the influence matrix $\mathbf{B}_{\mathbf{F}}$ is expressable as

$$B_{F} = M \Phi_{P} \tilde{B} \qquad (2-10)$$

Where \tilde{B} is an N \times m matrix and M denotes the L \times L positive definite mass matrix of the LSS, then

$$\phi_{\mathbf{R}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} = 0 \tag{2-11}$$

With such an influence matrix, the primary control-influence matrix is \tilde{B} , i.e.,

$$\phi_{\mathbf{p}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} = \tilde{\mathbf{B}} \tag{2-12}$$

Proof:

Since the mode shapes ϕ_1 are orthonormal with respect to the mass matrix by definition, $\phi_R^T M \phi_P = 0$ and $\phi_P^T M \phi_P = I$. This fact combined with (2-10) immediately results in (2-11) and (2-12).

It follows from this theorem that complete elimination of control spillover is possible, provided the special form of influence matrix expressed by (2-10) is realizable for some nontrivial matrix \tilde{B} through proper placement of actuators. And, depending on the matrix \tilde{B} , complete control of the primary modes is also possible; this can be seen from (2-12).

Equation (2-10) expresses an idealized situation: the elimination of control spillover to neglected modes requires no knowledge of these modes. Actually, it is rarely possible to place the actuators according only to the primary mode shapes for eliminating control spillover; information on the non-primary mode shapes is at least useful. However, computational experience with finite-element methods indicates that only the first half, or less, of the modeled modes can be accurately calculated; complete elimination of control spillover to all nonprimary modes is not really practical. In addition, real structures in general may not have locations for realizing such a special form of influence matrix, even actuators are allowed to be placed anywhere without any constraints. Structural constraints further reduces the possibility of eliminating control spillover to all the nonprimary modes. Hence, to be practical, one should only try to eliminate control spillover to some judiciously selected nonprimary modes.

Let $\{\omega_{Sj}, \phi_{Sj}\}$, $j=1,\ldots,M$, denote a selected subset of nonprimary modes; we call them secondary modes for a self-explanatory reason. Then the finite-element model (2-1)-(2-2) is now partitioned into three parts, instead, as follows.

$$\ddot{\eta}_{p} + 2Z_{p}\Omega_{p}\dot{\eta}_{p} + \Omega_{p}^{2}\eta_{p} = \Phi_{p}^{T}B_{F}u \qquad (2-13)$$

$$\ddot{\eta}_S + 2Z_S \Omega_S \dot{\eta}_S + \Omega_S^2 \eta_S = \phi_S^T B_F u \qquad (2-14)$$

$$\ddot{\eta}_R + 2Z_R \Omega_R \dot{\eta}_R + \Omega_R^2 \eta_R = \Phi_R^T B_F u \qquad (2-15)$$

$$y = (C_{D}^{\phi}_{P}^{\eta}_{P} + C_{V}^{\phi}_{P}^{\eta}_{P}) + (C_{D}^{\phi}_{S}^{\eta}_{S} + C_{V}^{\phi}_{S}^{\eta}_{S}) + (C_{D}^{\phi}_{R}^{\eta}_{R} + C_{V}^{\phi}_{R}^{\eta}_{R})$$
(2-16)

with $\eta = (\eta_p, \eta_S, \eta_R)$, where subscript "S" denotes the M secondary modes, whereas subscript "R" now denotes the residual L-M-N modes. For the remainder of this paper we shall discuss the elimination of control spillover to these M secondary modes, and the reduction, instead of elimination, of spillover to residual L-M-N modeled modes.

^{*} Also, see Section 2.5 for discussions on their selection.

Basically, to eliminate control spillover to the secondary modes is to find an influence matrix $\mathbf{B}_{\mathbf{p}}$ such that

$$\Phi_{\mathbf{S}}^{\mathbf{T}}\mathbf{B}_{\mathbf{F}} = 0 \tag{2-17}$$

Theorem 2:

(a) An influence matrix B_F satisfies Eq. (2-17) if and only if it is given by

$$B_{F} = (-Q_{1}W_{1}^{-1}W_{2} + Q_{2})\hat{B}$$
 (2-18)

where \hat{B} is an arbitrary (L-M) × m matrix, $Q = [Q_1,Q_2]$ is a nonsingular transformation, and W_1 is a nonsingular M × M matrix; W_1 and W_2 result from column operations (represented by Q) on Φ_S^T , namely

$$[W_1, W_2] = \Phi_S^{\mathrm{T}} Q$$
 (2-19)

(b) With such an influence matrix, the primary control-influence matrix is

$$\phi_{\mathbf{p}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} = (\mathbf{V}_{1} \mathbf{W}_{1}^{-1} \mathbf{W}_{2} + \mathbf{V}_{2}) \hat{\mathbf{B}}$$
 (2-20)

and is nonzero for some nonzero $\hat{B},$ where \textbf{V}_1 and \textbf{V}_2 result from the same column operations on $\boldsymbol{\Phi}_p^T,$ namely

$$[v_1, v_2] = \phi_p^T Q \qquad (2-21)$$

Proof:

(a) Sufficiency of (2-18) follows immediately from substitution of (2-18) and (2-19) in the left-hand side of (2-17). To prove the necessity, let Q and W₁ be any such matrices and rewrite Eq. (2-17) using (2-19) as follows

$$o = \phi_S^T B_F = \phi_S^T Q Q^{-1} B_F = [w_1, w_2] \begin{bmatrix} \bar{B} \\ \hat{B} \end{bmatrix} = w_1 \bar{B} + w_2 \hat{B}$$

From which it is necessary that, for any $\hat{\mathbf{B}}$

$$\bar{\mathbf{B}} = -\mathbf{W}_1^{-1}\mathbf{W}_2\hat{\mathbf{B}}$$

Since

$$Q^{-1}B_{F} = \begin{bmatrix} \widehat{B} \\ \widehat{B} \end{bmatrix} = \begin{bmatrix} -W_{1}^{-1}W_{2} \\ I \end{bmatrix} \widehat{B}$$

we must have

$$B_{F} = Q \begin{bmatrix} -W_{1}^{-1}W_{2} \\ I \end{bmatrix} \hat{B} = (-Q_{1}W_{1}^{-1}W_{2} + Q_{2})\hat{B}$$

as was to be proved.

(b) The expression (2-20) follows directly from (2-18) and (2-21). Assume on the contrary that the control-influence matrix thus given is identically zero for any \hat{B} . Then, (2-20) implies that $V_2 = V_1 W_1^{-1} W_2$. Consequently

$$\phi_{P}^{T}Q = [V_{1}, V_{2}] = V_{1}W_{1}^{-1}[W_{1}, W_{2}] = V_{1}W_{1}^{-1}\phi_{S}^{T}Q$$

Since Q is nonsingular, it follows that

$$\Phi_{\mathbf{P}}^{\mathbf{T}} = \mathbf{v}_{1} \mathbf{w}_{1}^{-1} \Phi_{\mathbf{S}}^{\mathbf{T}}$$

which is a contradiction to the linear independence among the mode shapes.

This theorem clearly indicates that elimination of control spillover to all the secondary modes is possible, provided that actuators can be properly placed to yield an influence matrix $B_{\rm F}$ as given by (2-18) for some Q and B. Note that all the (L-M)m elements of $\hat{\rm B}$ in (2-18) are adjustable. In most cases, the transformation Q in (2-18) is not unique; more freedom in adjusting the special form or the numerical value for $B_{\rm F}$ is thus provided.

It also indicates via (2-20) that complete control of the primary modes can be possible while elimination of control spillover to secondary modes is being realized by proper placement of actuators. To see it, consider a convenient choice of column operations that make the portion W_2 zero. (A Gaussian elimination process with partial pivoting on rows of Φ^T_S is one such choice.) The following is a refinement of Theorem 2 in this regard.

Theorem 3:

Suppose B is a desirable primary control-influence matrix. Let Q = [Q_1,Q_2] be a nonsingular L \times L transformation such that $\varphi_S^TQ_1$ is nonsingular and $\varphi_S^TQ_2$ is null. Then, an influence matrix B_F satisfies

$$\phi_{\mathbf{p}}^{\mathbf{T}}\mathbf{B}_{\mathbf{F}} = \tilde{\mathbf{B}} \tag{2-22}$$

and Eq. (2-17) simultaneously if, and only if, it is given by

$$\mathbf{B}_{\mathbf{F}} = \mathbf{Q}_{2} (\phi_{\mathbf{p}}^{\mathbf{T}} \mathbf{Q}_{2})^{\dagger} \tilde{\mathbf{B}} + \mathbf{Q}_{2} \mathbf{B}^{\mathbf{O}}$$
 (2-23)

where the superscript + denotes the (right) generalized inverse, namely

$$(\phi_{pQ_{2}}^{T})^{\dagger} = (\phi_{pQ_{2}}^{T})^{T} [(\phi^{T}Q_{2})(\phi^{T}Q_{2})^{T}]^{-1}$$
(2-24)

and B^{O} is an arbitrary (L-M) \times m matrix such that

$$(\phi_{\mathbf{p}}^{\mathbf{T}}Q_{2})\mathbf{B}^{\mathbf{O}} = 0 \tag{2-25}$$

Proof:

The sufficiency follows directly from substitution of (2-23) together with (2-24) and (2-25) in (2-17) and (2-22). To prove the necessity, note that a solution of (2-17) must be given by (2-18), namely

$$B_{F} = [Q_{1}, Q_{2}] \begin{bmatrix} 0 \\ I \end{bmatrix} \hat{B} = Q_{2} \hat{B}$$
 (2-26)

where $W_2 = 0$ for such a transformation Q. Equation (2-22) therefore becomes

$$\phi_{\mathbf{p}}^{\mathbf{T}} Q_{2} \hat{\mathbf{B}} = \tilde{\mathbf{B}} \tag{2-27}$$

To solve (2-27) for \hat{B} , we first claim that the N × (L-M) matrix $\Phi_p^TQ_2$ is of rank N. The composite (M+N) × (L-M) matrix

$$\begin{bmatrix} \phi_{S}^{T} \\ \phi_{P}^{T} \end{bmatrix} = [\phi_{S}, \phi_{P}]^{T}$$

must have rank (M+N), so must

$$\begin{bmatrix} \boldsymbol{\Phi}_{\mathbf{S}}^{\mathbf{T}} \\ \boldsymbol{\Phi}_{\mathbf{P}}^{\mathbf{T}} \end{bmatrix} \mathbf{Q} = \begin{bmatrix} \boldsymbol{\Phi}_{\mathbf{S}}^{\mathbf{T}} \mathbf{Q}_{1} & \mathbf{0} \\ \boldsymbol{\Phi}_{\mathbf{P}}^{\mathbf{T}} \mathbf{Q}_{1} & \boldsymbol{\Phi}_{\mathbf{P}}^{\mathbf{T}} \mathbf{Q}_{2} \end{bmatrix}$$

Consequently, $Q_p^TQ_2$ must have rank N, since $\phi_S^TQ_1$ has rank M. Therefore, the product $(\phi_p^TQ)(\phi_p^TQ)^T$ is nonsingular, and the (right) generalized inverse (2-24) is well defined. Moreover, a solution to Eq. (2-27) therefore exists, and is given by

$$\hat{\mathbf{B}} = (\Phi_{\mathbf{p}}^{\mathbf{T}} \mathbf{Q}_{2})^{\dagger} \tilde{\mathbf{B}} + \mathbf{B}^{\mathbf{O}}$$

where B^{O} is an arbitrary matrix satisfying (2-25). Substitution of this relationship in (2-26) completes the proof of the necessity of relationship (2-23).

Theorem 3 clearly implies that if the m actuators can be so placed that the influence matrix B_F expressed by (2-23) is realized for some matrices B, $B^{\rm O}$, and Q, which are all adjustable, then control spillover to the M secondary modes can be eliminated without sacrificing complete control of the N primary modes, and vice versa. Thus, Eq. (2-23) can serve as a guide for placement of actuators for the dual purposes: start with some B, $B^{\rm O}$, Q; examine the resultant influence matrix computed through (2-23) to see if it is realizable; vary B and $B^{\rm O}$, even Q, if it is not realizable. Obviously, if exact realization is not possible, close realization is still very desirable; control spillover, though not eliminated, is greatly reduced.

Before concluding this section, let us remark that Theorem 1 becomes a specialization of Theorem 3: requiring that all nonprimary modes be the secondary modes (i.e., that M = L-N), and setting $Q_1 = M\Phi_S^T$, $Q_2 = M\Phi_P^T$, and $B^O = 0$ specializes (2-23) to (2-10).

2.3 Synthesis of Actuator Influences

In general, actuator locations on the structure are constrained; influence matrix B_F expressed by (2-10), or (2-23), or even (2-18) may not be completely realized, hence control spillover may still be serious. On the other hand, actuator locations may have been predetermined, or the actuators may have already been placed; control spillover most likely will exist. Elimination of control spillover must then be accomplished by synthesizing the existent influences of the actuators on the structure. The main idea is to conceptually replace the m physical actuators by m' "synthetic actuators". Denote by B_F^* the influence matrix of the synthetic actuators and v the m'-vector of their inputs. Then we have

$$\Phi^{T}B_{F}u = \Phi^{T}B_{F}v$$

Control spillover to secondary modes now becomes $\Phi_S^T B_F^{\dagger} v$. Thus, if B_F^{\dagger} can be so chosen that $\Phi_S^T B_F^{\dagger} = 0$, then control spillover to the secondary modes can be eliminated.

Consider the simplest kind of synthetic actuators formed by linear combination of the influences of individual physical actuators, namely

$$B_{F}^{\dagger} = B_{F}^{\Gamma}$$

where Γ is an m × m' transformation to be determined. (See Figure 2-2 for illustration.) Given an influence matrix B_F , to eliminate control spillover to the secondary modes is thus to find an m'× m' matrix Γ such that

$$\Phi_{\mathbf{S}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} \mathbf{\Gamma} = 0 \tag{2-28}$$

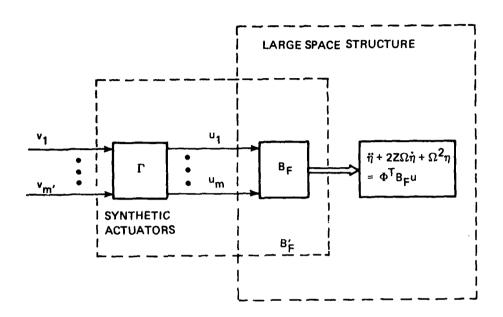


Figure 2-2. Synthesis of control influences.

Theorem 4:

(a) A nontrivial transformation Γ satisfying (2-28) exists if, and only if

$$r \stackrel{\triangle}{=} rank(\phi_S^T B_F) < m$$
 (2-29)

(b) Assume r < m. Then a transformation Γ satisfies (2-26) if, and only if, it is given by

$$\Gamma = (-Q_1 W_{11}^{-1} W_{12} + Q_{12}) \hat{\Gamma}$$
 (2-30)

where Γ is an arbitrary (m-r) \times m' matrix, Q = [Q_1,Q_2] a nonsingular m \times m transformation, and W_1 a nonsingular r \times r matrix; Q_1 and Q_2 have dimension m \times r and m \times (m-r) respectively; W_1 and W_12 result from column operations (represented by Q) and row rearrangement (represented by nonsingular M \times M transformation P) on the product $\Phi^{\rm TS}_{\rm F}_{\rm F}$, namely

$$\begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} \\ \mathbf{W}_{21} & \mathbf{W}_{22} \end{bmatrix} = \mathbf{P}(\Phi_{\mathbf{S}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}}) \mathbf{Q}$$
 (2-31)

Proof:

(a) To prove the necessity of condition (2-29), assume on the contrary that $r \ge m$. Then $r = m \le M$, since the product $\Phi_S^TB_F$ is of dimension $M \times m$. This implies that $\Gamma = 0$ is the only possible solution to (2-28). To prove its sufficiency, presume the validity of the general expression (2-30), and claim that Γ cannot be identically zero. Suppose on the contrary that it is identically zero for any $\hat{\Gamma}$. This implies that

$$Q_2 = Q_1 W_{11}^{-1} W_{12}$$

which is a contradiction to the nonsingularity of the transformation $Q = [Q_1, Q_2]$.

(b) Now, we prove the sufficiency of (2-30). With (2-31), we have

Since rank(W_{11}) = r = rank($\Phi_S^T B_F$) = rank($P\Phi_S^T B_F Q$), it follows* that

$$W_{22} = W_{21}W_{11}^{-1}W_{12} \tag{2-32}$$

Therefore, Eq. (2-28) is satisfied. To prove the necessity, rewrite Eq. (2-28) using such nonsingular transformations Q and P as follows

$$0 = \Phi_{\mathbf{S}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} \Gamma = \mathbf{P}^{-1} \mathbf{P} \Phi_{\mathbf{S}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} Q Q^{-1} \Gamma = \mathbf{P}^{-1} \begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} \\ \mathbf{W}_{21} & \mathbf{W}_{22} \end{bmatrix} \begin{bmatrix} \bar{\Gamma} \\ \hat{\Gamma} \end{bmatrix}$$

^{*} See Reference 7 page 47.

where (2-31) was used. It is therefore necessary that

$$W_{11}\bar{r} + W_{12}\hat{r} = 0$$
 (2-33)

$$W_{21}\bar{\Gamma} + W_{22}\hat{\Gamma} = 0 ag{2-34}$$

But, in view of the equality (2-32), Eq. (2-34) is nothing but a linear transformation of Eq. (2-33); satisfaction of (2-33) implies that of (2-34). Now from (2-33), we must have

$$\vec{\Gamma} = -W_{11}^{-1}W_{12}\hat{\Gamma}$$

for any $\hat{\Gamma}$. Then by definition

$$\mathbf{Q}^{-1}\mathbf{\Gamma} = \begin{bmatrix} \overline{\Gamma} \\ \\ \widehat{\Gamma} \end{bmatrix} = \begin{bmatrix} -\mathbf{W}_{11}^{-1}\mathbf{W}_{12}\widehat{\Gamma} \\ \\ \widehat{\Gamma} \end{bmatrix}$$

Therefore,

$$\Gamma = Q \begin{bmatrix} -W_{11}^{-1}W_{12} \\ 1 \end{bmatrix} \hat{\Gamma} = (Q_1W_{11}^{-1}W_{12} + Q_2)\hat{\Gamma}$$

Theorem 5:

The synthesized primary control-influence matrix with the transformation Γ given by (2-30) is

$$\phi_{\mathbf{p}}^{\mathbf{T}} \mathbf{B}_{\mathbf{p}} \Gamma = (-\mathbf{V}_{1} \mathbf{W}_{1:1}^{-1} \mathbf{W}_{1:2} + \mathbf{V}_{2}) \hat{\Gamma}$$
 (2-35)

where V_1 and V_2 result from the same column operations on the product $\Phi_p^T B_F$, namely

$$[V_1, V_2] = \Phi_P^T B_F [Q_1, Q_2]$$
 (2-36)

It is nonzero for some nonzero $\hat{\Gamma}$ if and only if no row of $\Phi_p^TB_F$ is a linear combination of rows of $\Phi_S^TB_F$, namely, if and only if

$$\operatorname{rank}\left(\begin{bmatrix} \phi_{S}^{T} B_{F} \\ \phi_{P}^{T} B_{F} \end{bmatrix}\right) \geq \operatorname{rank}(\phi_{S}^{T} B_{F}) + 1 \qquad (2-37)$$

Proof:

Equations (2-30), (2-31), and (2-36) yield (2-35) directly

Now, assume the matrix given by (2-34) is zero for any $\hat{\Gamma}$. Then

$$v_2 = w_1 w_{11}^{-1} w_{12}$$

Therefore, (2-36) becomes

$$\Phi_{\mathbf{p}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} \mathbf{Q} = \mathbf{v}_{1} \mathbf{w}_{11}^{-1} [\mathbf{w}_{11}, \mathbf{w}_{12}] = [\mathbf{v}_{1} \mathbf{w}_{11}^{-1}, \mathbf{Y}] \begin{bmatrix} \mathbf{w}_{11} & \mathbf{w}_{12} \\ & & \\ \mathbf{w}_{21} & \mathbf{w}_{22} \end{bmatrix} = [\mathbf{v}_{1} \mathbf{w}_{11}^{-1}, \mathbf{Y}] \mathbf{p} \Phi_{\mathbf{S}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} \mathbf{Q}$$

where Y is any N \times (M-r) matrix such that YW₂₁ = 0. Postmultiplying by Q⁻¹ yields

$$\phi_{\mathbf{P}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} = [\mathbf{V}_{1} \mathbf{W}_{11}^{-1}, \mathbf{Y}] \mathbf{P} \phi_{\mathbf{S}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}}$$

This implies that rows of $\Phi_P^TB_F$ are but linear combinations of rows of $\Phi_P^TB_F$. This proves the sufficiency of condition (2-37). To prove its necessity, assume on the contrary that at least one row of $\Phi_P^TB_F$ is a linear combination of rows of $\Phi_S^TB_F$. Then, for some nontrivial N × M matrix T, we can write

$$\Phi_{\mathbf{P}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} = \mathbf{T} \Phi_{\mathbf{S}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}}$$

Consequently, the synthesized control-influence matrix is

$$\begin{split} \Phi_{\mathbf{P}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} \Gamma &= \mathbf{T} \Phi_{\mathbf{S}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} \Gamma &= \mathbf{T} \Phi_{\mathbf{S}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} (-\mathbf{Q}_{1} \mathbf{W}_{11}^{-1} \mathbf{W}_{12} + \mathbf{Q}_{2}) \hat{\Gamma} \\ &= \mathbf{T} \mathbf{P}^{-1} \mathbf{P} \Phi_{\mathbf{S}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} \mathbf{Q} \begin{bmatrix} -\mathbf{W}_{11}^{-1} \mathbf{W}_{12} \\ \mathbf{I} \end{bmatrix} \hat{\Gamma} \\ &= \mathbf{T} \mathbf{P}^{-1} \begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} \\ \mathbf{W}_{21} & \mathbf{W}_{22} \end{bmatrix} \begin{bmatrix} -\mathbf{W}_{11}^{-1} \mathbf{W}_{12} \\ \mathbf{I} \end{bmatrix} \hat{\Gamma} &= \mathbf{0} \end{split}$$

for any $\hat{\Gamma}$, where the quality (2-32) was used. This contradiction proves the necessity of the stated condition.

The rank of M × m matrix $\phi_S^T B_F$ represents the equivalent number of independent secondary modes viewed from the input terminals, and hence the minimum number of physical actuators required for eliminating control spill-over to all M secondary modes. Similarly, the rank of matrix

$$\begin{bmatrix} \Phi_{\mathbf{S}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} \\ \Phi_{\mathbf{P}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} \end{bmatrix}$$

represents the minimum number of physical actuators required for both eliminating control spillover to the secondary modes, and for controlling the primary modes. Note that condition (2-37) implies that the actuators must be able to influence primary modes independently of secondary modes. Conditions (2-29) and (2-37) together mean that the number m of physical actuators required is at least

$$rank(\phi_S^T B_F) + 1$$

Consequently, placement of the actuators proper for reducing the number of input-equivalent independent secondary modes will help reduce the required number of actuators.

It follows immediately that if the M \times m matrix ${}^{\varphi}_S^T B_F^{}$ has rank M, then at least M + 1 physical actuators must be used.

Using the conditions established in Reference 5, for complete controllability of primary modes, one can shown that complete controllability of the primary modes by such m' synthetic actuators requires that the primay modes be completely controllable by the m physical actuators. The minimum numbers of

physical and synthetic actuators required for complete controllability of the primary modes in conjunction with elimination of control spillover to the secondary modes are

m
$$\geq$$
 rank($\phi_S^T B_F$) + max{multiplicity of ω_{Pi} }
$$1 \leq i \leq N$$
(2-38)

m'
$$\geq \max\{\text{multiplicity of } \omega_{\text{Pi}}\}$$
 (2-39) $1 \leq i \leq N$

For condition (2-38), one can easily see that for a fixed number m of physical actuators, reduction in the number M of secondary modes, or alternatively the rank r of control-spillover matrix $\phi^T_S B_F$, will alleviate the difficulty in complete control of primary modes while eliminating control spillover to secondary modes.

The validity of Theorems 4 and 5 is independent of the number m' of synthetic actuators. Hence, m' can be chosen for any purpose. For example, in applying Davison's method of pole assignment to model (2-3)-(2-4) when m > N, the m actuators must be synthesized (combined or reduced) to form at most N synthetic actuators, namely, m' \leq N. Undoubtedly, (2-39) should be satisfied in choosing m' for any case.

Example

Consider the Draper tetrahedral structure (see Appendix A). Six actuators already have been placed. Choose modes 1, 2, 4, and 5 (the four line-of-sight critical modes) as the primary modes and, to the extreme, take all the remaining known modeled modes (i.e., modes 3, 6, 7, and 8) as the secondary modes. This is to examine whether it is possible, by synthesizing the given actuator influences, to control the four primary modes without spillover to all nonprimary modes. Note that in this example, L=8, N=4, M=4, and m=6. Listed in the order of 3-6-7-8 (the four secondary modes) and 1-2-4-5 (the four primary modes), the control-spillover matrix $\phi_{\rm S}^{\rm T}$ and the control-influence matrix $\phi_{\rm T}^{\rm T}$ are given as follows

Note that the control-spillover matrix has the same order of magnitude as the control-influence matrix (in fact, elements of $\phi_S^T B_F$ have a magnitude between 0.046 and 0.369, while those of $\phi_P^T B_F$ have only between 0.017 and 0.351). Control spillover from the physical actuators is significant. To eliminate it by synthesizing the actuator influences, perform column operations on both matrices by Gaussian elimination and get

_					_
<u> </u>	0	0	0	i 0	0
-1.063	1	0	0	, 0	0
1.360	-1.278	1	0	1 0	0
-1.102	0	-0.653	1	1 0	0
		-			
0.247	-0.232	0.131	0	1 1	0
0.062	0	-0.155	0.238	-0.668	1
0.219	-0.207	-0.188	0	6.869	-0.001
0.179	0	-0.206	0.350	7.053	-10.565
_				1	_
C 0	0	0	0	1.776	-13.294
0	0	0	0	1 0	-13.294
-3.685	1.732	-0.933	0.317	1-4.327	1.246
0	0	-1.451	1.110	7.742	-3.541
0	0	0	1.110	-3.018	-3.541
Lo	-1.732	0.518	0.317	1 2.668	1.248_
	1.360 -1.102 0.247 0.062 0.219 0.179 0 0 -3.685 0	-1.063 1 1.360 -1.278 -1.102 0 0.247 -0.232 0.062 0 0.219 -0.207 0.179 0 0 0 -3.685 1.732 0 0 0 0	$ \begin{bmatrix} -1.063 & 1 & 0 \\ 1.360 & -1.278 & 1 \\ -1.102 & 0 & -0.653 \\ \hline -2.0.247 & -0.232 & 0.131 \\ 0.062 & 0 & -0.155 \\ 0.219 & -0.207 & -0.188 \\ 0.179 & 0 & -0.206 \\ \hline \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -3.685 & 1.732 & -0.933 \\ 0 & 0 & -1.451 \\ 0 & 0 & 0 \\ \end{bmatrix} $	$ \begin{bmatrix} -1.063 & 1 & 0 & 0 \\ 1.360 & -1.278 & 1 & 0 \\ -1.102 & 0 & -0.653 & 1 \\ -2.000 & -0.653 & 1 \\ -2.000 & -0.232 & 0.131 & 0 \\ 0.062 & 0 & -0.155 & 0.238 \\ 0.219 & -0.207 & -0.188 & 0 \\ 0.179 & 0 & -0.206 & 0.350 \\ \end{bmatrix} $	-1.063

Note that both conditions (2-29) and (2-37) are satisfied; control of the four primary modes, as well as elimination of control spillover to the four secondary modes, is possible by synthesizing the influences of the six already placed actuators. All we need now is to determine a 2 × m' matrix $\hat{\Gamma}$ so that a desired transformation Γ can be computed from (2-30), noting that $W_{12} = W_{21} = W_{22} = 0$ with such column operations. First, Eq. (2-35) thereby becomes

$$\phi_{\mathbf{P}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} \Gamma = \mathbf{v}_{2} \hat{\Gamma} = \begin{bmatrix} 1 & 0 \\ -0.668 & 1 \\ 6.869 & -0.001 \\ 7.053 & -10.565 \end{bmatrix} \hat{\Gamma}$$

Second, the number m' of synthetic actuators is only required to be at least 1, according to condition (2-39), since all modes have distinct natural frequencies. For demonstration, compute first for the case with m' = 1. The resultant synthesized control-influence matrix is a vector of four components. Arbitrarily let the desired first and second components be 1 and -1, namely

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -0.668 & 1 \end{bmatrix} \hat{\mathbf{r}}$$

From which we get $\hat{\Gamma}$, and hence Γ and $\Phi_p^T B_p T$ as follows

$$\hat{\Gamma} = \begin{bmatrix} 1 \\ -0.332 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 6.190 \\ 4.414 \\ -4.471 \\ 8.918 \\ -1.842 \\ 2.254 \end{bmatrix}, \quad \Phi_{P}^{T}B_{F}\Gamma = \begin{bmatrix} 1 \\ -1 \\ 6.869 \\ 10.561 \end{bmatrix}$$

From the resultant synthesized control-influence matrix (or more precisely, vector) above, we can see that a single input (to a single synthetic actuator) is enough. Nevertheless, compute also for the case with $\mathbf{m'}=2$. Again, arbitrarily let the desired first and second rows of the synthesized control-influence matrix be [1,0] and [0,1] respectively, namely

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -0.668 & 1 \end{bmatrix} \hat{\Gamma}$$

Then, from which we get the following $\hat{\Gamma}$, Γ and $\Phi_p^TB_F\Gamma$

$$\hat{\Gamma} = \begin{bmatrix} 1 & 0 \\ 0.668 & -1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} -7.014 & 13.294 \\ -8.880 & 13.294 \\ -3.495 & -1.246 \\ 5.377 & 3.541 \\ -5.383 & 3.541 \\ 3.502 & -1.248 \end{bmatrix}, \quad \Phi_{\mathbf{P}}^{\mathbf{T}} \mathbf{B}_{\mathbf{F}} \Gamma = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 6.868 & 0.001 \\ -0.004 & 10.565 \end{bmatrix}$$

From the resultant synthesized control-influence matrix, we can see that a double input (one each synthetic actuator) is also enough. It is not clear, however, if it is advantageous to use more synthetic actuators than necessary, judging from the relative magnitude of the elements in Γ for both cases: these elements may be interpreted as required amplifier gains.

2.4 Compensation of Actuator Inputs

Control spillover can be reduced, though not completely eliminated, by proper shaping of the actuator inputs. The idea is to insert a compensator (or filter) G (s) to each input channel so that frequency components other than those of primary modes will not pass without attenuation. Spillover to virtually all nonprimary modes can be reduced this way, although the reduction may not be uniformly significant. Compensators can be used without

having to properly place the actuators or properly synthesize the actuator influences first; but, proper placement and synthesis certainly will make spillover reduction by compensation easier and more effective. For generality, the influence matrix B_F in this section may denote either that of physical actuators or that of synthesized actuators.

Let the following denote the generic frequency component of the $\boldsymbol{k}^{\mbox{th}}$ input

$$u_k(t) = \alpha \cos \omega t + \beta \sin \omega t$$
 (2-40)

where the amplitude $\rho = \sqrt{\alpha^2 + \beta^2}$ and the phase $\theta = \tan^{-1} \beta/\alpha$ may vary with the generic frequency ω . Consider the response of the ith mode $\{\omega_i, \phi_i\}$ to kth input alone. From (2-1), we have

$$\ddot{\eta}_{i}(t) + 2\zeta_{i}\omega_{i}\dot{\eta}(t) + \omega_{i}^{2}\eta_{i}(t) = \phi_{i}^{T}b_{Fk}u_{k}(t) = \phi_{i}^{T}b_{Fk}(\alpha \cos \omega t + \beta \sin \omega t)$$
(2-41)

where b_{Fk} denotes the k^{th} column of matrix B_F . Taking the Laplace transform of (2-40) and (2-41) yields

$$H_{i}(s) = \phi_{i}^{T} b_{Fk} \frac{\alpha s + \beta \omega}{(s^{2} + 2\zeta_{i}\omega_{i}s + \omega_{i}^{2})(s^{2} + \omega^{2})}$$
(2-42)

By partial-fraction expansion

$$H_{i}(s) = \frac{\phi_{i}^{T}b_{Fk}}{(\omega^{2} - \omega_{i}^{2})^{2} + 4\zeta_{i}^{2}\omega_{i}^{2}\omega^{2}} \left[\frac{as + b}{s^{2} + 2\zeta_{i}\omega_{i}s + \omega_{i}^{2}} + \frac{cs + d\omega}{s^{2} + \omega^{2}} \right]$$

Where

$$-c = a = (\omega^2 - \omega_1^2)\alpha + 2\zeta_1\omega_1\omega\beta$$

$$b = -2\zeta_1\omega_1^3\alpha + (\omega^2 - \omega_1^2 + 4\zeta_1^2\omega_1^2)\omega\beta$$

$$d = 2\zeta_1\omega_1\omega\alpha - (\omega^2 - \omega_1^2)\beta$$

Taking the inverse Laplace transform gives the time-domain response

$$\eta_{i}(t) = \phi_{i}^{T} b_{Fk}^{T} p e^{-\zeta_{i} \omega_{i} t} \sin \left(\omega_{i} \sqrt{1 - \zeta_{i}^{2}} t + \psi \right)$$

$$+ \phi_{i}^{T} b_{Fk}^{T} q \sin (\omega t + \xi) \qquad (2-43)$$

where

$$p = \left(\frac{\omega_{i}^{2}\alpha^{2} - 2\zeta_{i}\omega_{i}\omega\alpha\beta + \omega^{2}\beta^{2}}{\omega_{i}^{2}\left(1 - \zeta_{i}^{2}\right)\left[\left(\omega^{2} - \omega_{i}^{2}\right)^{2} + 4\zeta_{i}^{2}\omega_{i}^{2}\omega^{2}\right]}\right)^{1/2}$$

$$\psi = \tan^{-1}\frac{-\zeta_{i}\omega_{i}\left(\omega^{2} + \omega_{i}^{2}\right)\alpha + \left(\omega^{2} - \omega_{i}^{2} + 2\zeta_{i}^{2}\omega_{i}^{2}\right)\omega\beta}{\omega_{i}\sqrt{1 - \zeta_{i}^{2}}\left[\left(\omega^{2} - \omega_{i}^{2}\right)\alpha + 2\zeta_{i}\omega_{i}\omega\beta\right]}$$

$$q = \left(\frac{\alpha^{2} + \beta^{2}}{\left(\omega^{2} - \omega_{i}^{2}\right)^{2} + 4\zeta_{i}^{2}\omega_{i}^{2}\omega^{2}}\right)^{1/2}$$

$$\xi = \tan^{-1}\frac{2\zeta_{i}\omega_{i}\omega\alpha - \left(\omega^{2} - \omega_{i}^{2}\right)\beta}{-\left(\omega^{2} - \omega_{i}^{2}\right)\alpha - 2\zeta_{i}\omega_{i}\omega\beta}$$

Now assume that mode i is a residual mode and that $\phi_1^T b_{Fk} \neq 0$. Then, to reduce the spillover of control input $u_k(t)$ to mode i is to reduce the magnitude of its response $\eta_1(t)$. Note first that since ζ_1 is negligibly small, the extremely slow decaying natural oscillation (with frequency $\omega_1\sqrt{1-\zeta_1}$) cannot, and should not, be ignored in the practical sense, especially for the case where vibration needs to be settled down within minutes or even seconds after slewing the flexible structure. In particular, when the damping $\zeta_1\omega_1$ is virtually zero, the corresponding settling time is virtually infinite and the natural oscillation will (like the steady-state forced oscillation) last forever. Furthermore, it has almost the same magnitude as the forced oscillation: for the sinusoidal input with $\beta=0$, magnitude p is slightly larger than magnitude q for all input frequency ω ; for the input with $\alpha=0$, magnitude p is much larger than magnitude q for high input frequency ω and the ratio p/q increases with ω .

Now, with $\alpha = \rho \cos \theta$ and $\beta = \rho \sin \theta$, it is easy to see that both magnitudes p and q decrease with amplitude ρ of the sinusoidal input. Moreover, examination of the dependence of magnitudes p and q on the input frequency ω easily shows that the frequency spectrum of response $n_i(t)$ spans the entire frequency domain. Thus, an obvious way for reducing control spillover to mode i is to attenuate all frequency components of the control input. It is an impractical way, however, since certain frequency components must be sufficiently large, instead, for controlling the primary modes. With control influence $\Phi_{1}^{T}b_{Fk}$, the damping ration ζ_{1} , and the natural frequency ω_{1} all fixed, the impossibility of complete elimination of control spillover while controlling the primary mode is clear.

Attenuation of control inputs must be frequency selective. Due to light damping, the peak magnitude of p and q at the resonant frequencies are the basic concern in compensating control inputs. Consider the magnitude q first.

It reaches maximum at $\omega = \omega_1 \sqrt{1-2\zeta_1^2}$ with $q_{max} = \rho/(2\omega_1^2\zeta_1\sqrt{1-\zeta_1^2})$. Thus, the compensator must at least be able to provide sufficient attenuation to offset the peak magnitude q_{max} at frequency $\omega_1\sqrt{1-2\zeta_1^2}$. At this point, it is worth pointing out that the larger the damping ratio ζ_1 is, the easier it is to attenuate the resonance peak. For example, $q_{max} = \infty$, for $\zeta_1 = 0$; $q_{max} = (10/\omega_1^2)\rho$, for $\zeta_1 = 0.05$; $q_{max} = (5/\omega_1^2)\rho$, for $\zeta_1 = 0.1$.

Next, consider the magnitude p. It reaches maximum also at $\omega = \omega_1 \sqrt{1-2\zeta_1^2}$ for $\beta=0$, but at $\omega=\omega_1$ for $\alpha=0$; both extreme cases have the same peak magnitude as q_{max} . Therefore, the compensator must be a bandstop filter to provide sufficient attenuation to a band of frequency components since, in general, the inputs do not assume the extreme cases. The band must contain both frequencies $\omega_1\sqrt{1-2\zeta_1^2}$ and ω_1 . It is worth mentioning that, even when ζ_1 is very small but nonzero, the difference between these two frequencies may be significant for high frequency ω_1 .

Now, with a compensator G (s) inserted in each control input channel, as shown in Figure 2-3, the Laplace transform of the ith modal response becomes

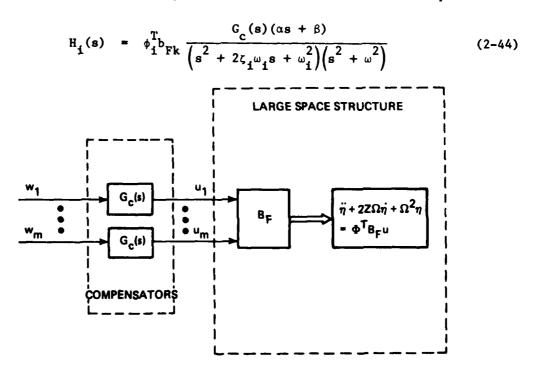


Figure 2-3. Compensation of control inputs.

Although it may sound more general or more flexible if a different compensator is inserted in each channel, there is no real advantage in doing so. For it is difficult to predetermine what frequency components will definitely not appear in a specific control channel, especially when the control inputs contain feedback of actual modal responses.

Compared with (2-42), it shows that the compensator has the equivalent effect of reshaping the (flat) control influence $\phi_1^T b_{Fk}$. For reducing control spill-over to residual modes while preserving the control of primary modes, the design of the compensator must satisfy the following principal requirements.

- (1) The real part of each pole of $G_c(s)$ be sufficiently negative.
- (2) The real part of each zero of $G_{c}(s)$ be nonpositive.
- (3) G (s) be a bandstop filter for each residual mode frequency component.
- (4) G (s) be a bandpass filter for each primary mode frequency compenent.

Filters of various kinds of general forms can be synthesized to provide (or to closely approximate) a desired frequency spectrum satisfying the above requirements. [8] However, separate or combined use of the following simple kinds of passive filters is worth considering: cascade bandpass filters with primary-mode frequencies ω_{Pi} as the centers; cascade bandstop filters with residual-mode frequencies ω_{Ri} as the centers; cascade bridge-T networks [9] with

$$-\zeta_{Ri}\omega_{Ri} \pm j\omega_{Ri}\sqrt{1-\zeta_{Ri}^2}$$

as zeros; low-pass filters; lag networks; lead-lag networks; lag-lag networks, etc.

2.5 Three-Step Combined Approach

Each of the three approaches discussed in the foregoing sections can be separately used to reduce control spillover, but a combined approach will be more effective. The following three-step approach is a logical and natural combination.

- <u>Step 1</u>: Elimination of control spillover to secondary modes by proper placement of actuators on the structure.
- <u>Step 2</u>: Further elimination of control spillover to secondary modes by proper synthesis of the actuator influences.
- <u>Step 3</u>: Reduction of control spillover to residual modes by proper compensation of the control inputs.

Figure 2-4 shows the combination of the three steps. The resultant synthesizer Γ and compensators $G_{\mathbf{C}}(s)$ are then included in the design of control inputs \mathbf{w}_1 , ..., \mathbf{w}_m . The control inputs can be open-loop feedforward slew commands or state-feedback controls. If these three steps are followed in the dual manner for reduction of observation spillover, the control inputs can also be designed as output-feedback controls.

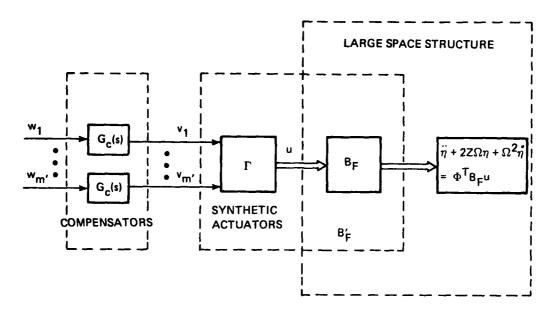


Figure 2-4. Three-step combined approach.

The following are some worthwhile remarks.

- (1) If compensators G (s) are added after control inputs or feed-back controllers are designed, the structure may be destabilized. For example, the addition of low-pass filters (or actuator dynamics), even of such a simple form as 1/(s + a), a > 0, can cause the phase to shift below -180° before the feed-back gains cross the 0 decibel line. In other words, all the compensators (including actuator/sensor dynamics) should be considered a part of the system model for control design.
- (2) Naturally, primary modes should contain those modes which are critical to the specific performance desired (say, the lineof-sight accuracy) of the structure. A judicious selection of noncritical modes may be included as primary modes to enhance the performance and stability of the controlled structure: for example, those which are susceptible to environmental or onboard disturbances, or highly sensitive to control spillover. The number N of primary modes need not be kept constant of control and design techniques. For example, more modes can be considered primary in direct output-feedback control than in state-feedback control with dynamic state estimation. In the design of output-feedback controllers, Canavin's method of modal decoupling can include more modes in the design model (2-3)-(2-4) because of simplicity in the required computations than Levine-Athan's method of optimal output feedback control.

- (3) The selection of noncritical modes for primary modes need not be independent of the selection for secondary modes. In particular, the selection should be duly adjusted to satisfy at least condition (2-37). For complete controllability of the primary modes, as well as elimination of control spillover to secondary modes via synthesis of actuator influences, it is even necessary to assure that no single row $\Phi_{\mathbf{r},\mathbf{r}}^{\mathbf{T}}$ of control influence on primary modes is a linear combination of rows $\Phi_{\mathbf{r},\mathbf{r}}^{\mathbf{T}}$ of control influence on secondary modes.
- (4) In Step 1, the secondary modes may be those important noncritical modes which cannot be included as primary modes for any reasons or those nonprimary modes which have natural frequencies identical to or closely spaced with some primary-mode frequency.
- (5) In Step 2, the secondary modes may be augmented by those non-primary modes having large magnitude $||\phi_1^T R_F||$ of control influence. Conversely, those which have negligibly small magnitude of control influence can be deleted from the set of secondary modes.
- (6) In Step 3, in case of an extremely large number of residual modes, the design of compensators should focus on those residual modes having larger magnitude $||\phi_{\bf i}^T B_F||$ of control influence, larger magnitudes p and q, smaller damping ratio $\zeta_{\bf i}$, or smaller natural frequency $\omega_{\bf i}$.

2.6 Conclusions

Control (observation) spillover may be reduced by proper placement of actuators (sensors). If proper placement is not possible because of structural constraints, control spillover to (observation spillover from) secondary modes can still be eliminated by synthesizing the actuator (sensor) influences. Alternatively, control spillover to (observation spillover from) nonprimary modes can be reduced, to various extent, by adding some compensation to each input (output) channel. A combined systematic use of placement, synthesis and compensation will yield better results.

LIST OF REFERENCES

- 1. Balas, M.J., "Active Control of Flexible Systems", <u>J. Optimization Theory and Applications</u>, Vol. 25, No. 3, pp. 415-436, July 1978; also in <u>Proc. of AIAA Symp. on Dynamics and Control of Flexible Spacecraft</u>, Blacksburg, VA, June 1977, pp. 217-236.
- Balas, M.J., "Modal Control of Certain Flexible Dynamic Systems", SIAM J. Control and Optimiz., Vol. 16, No. 3, pp. 450-462, May 1978.
- 3. Heffs, J., "The Effect of Erroneous Models on the Kalman Filter Response", IEEE Trans. Automat. Contr., Vol. AC-11, pp. 541-543, July 1966.
- 4. Nishimura, T., <u>Modeling Errors in Kalman Filters</u>, Technical Report 32-1319, Jet Propulsion Laboratory, Calif. Inst. of Technology, June 1970.
- 5. Lin, J.G., Y.H. Lin, D.R. Hegg, T.L. Johnson, and J.E. Keat, Theory of Design Methods, Interim Technical Report, Actively Controlled Structures Theory, R-1249, Volume 1, The Charles Stark Draper Laboratory, Inc., April 1979.
- 6. Sesak, J.R., "Control of Large Space Structures via Singular Perturbation Optimal Control", Paper No. 78-1690, AIAA Conf. on Large Space Platforms, Los Angeles, CA, September 17-19, 1978.
- 7. Gantmacher, F.R., The Theory of Matrices, Vol. I, Chelsea, New York, 1960.
- 8. Temes, G.C. and J.W. LaPatra, <u>Introduction to Circuit Synthesis and</u> Design, McGraw-Hill, New York, 1977.
- 9. Kuo, B.C., <u>Automatic Control Systems</u>, 3rd Ed., Prentice-Hall, Englewood Cliffs, N.J., 1975.

SECTION 3

OUTPUT FEEDBACK

3.1 Synopsis of Previous Research

A survey of several selected methods for the design of controllers with multivariable dynamic processes was recently reported. [1,2]* Since this section reports results arising out of the discoveries in that survey, a brief synopsis of Reference 1 is given to provide continuity.

The survey in Reference 1 focused on the following five design methods.

- Modal Decoupling [3] (1)
- Pole Assignment [4] (2)
- Optimal Output Feedback^[5] (3)
- Suboptimal Output Feedback [6] (4)
- Stochastic Optimal Output Feedback [7] (5)

These methods were selected as representative of the distinct approaches that have been proposed in the literature for control by constant-gain output feedback. The primary purpose of the survey was to evaluate the suitability of these methods for the problem of designing reduced-order controllers for multivariable processes of very high dimension. This problem inevitably arises in the consideration of active control for large space structures. For each method, the following was accomplished: survey of the relevant literature; investigation in depth into the theoretical basis for the method; discovery of new results (in some cases); and application of the method to controller design for one vibration mode of a specific two-mass oscillator. In addition, an overall comparative evaluation of the five methods was made, which included a detailed performance comparison of the various designs with the two-mass oscillator.

Distinguishing characteristics of the five design methods are briefly recalled. The plant adopted for controller design purposes is in each case a re 'ced-order finite-element model in normal mode coordinates embodying the structural modes considered critical for analysis of stability and control. Model development and notation is detailed in [1; Chap. 2]. The Modal Decoupling Method introduces artificial damping through the controller in such a way that the closed-loop equations for the critical modes are uncoupled from one another. Generalized inverses in the sense of Moore-Penrose[8,9] are used in designing the controller. The principal advantage of this method is that precise statements about stability of the complete high-order system embodying the reduced-order controller can be made. New stability results beyond those in Reference 3 were reported in [1; Sec. 3.5.3]. The principal disadvantage

Bracketed numerals refer to similarly numbered items in the List of References at the end of the section.

of the method is that stability guarantees assume colocation of sensors and actuators. The Pole Assignment Method is an algorithm for choosing controller gains so as to arrive at a closed-loop system in which as many of the system poles as possible lie at or near prespecified positions in the complex plane. The principal advantage of this method is that it provides a systematic way of precisely adjusting the dynamic characteristics of the closed-loop system. The principal disadvantage of this method is that, without certain restrictions, not all of the closed-loop poles in the design model can be "assigned", which may significantly affect the system performance. Certain specific suggestions for improving the method were identified [1; Sec. 4.2.3]. The Optimal Output Feedback Method is a generalization of the optimal state regulator design [10; Chap. 9] to the situation in which not all of the system states are available for measurement. The consequent wider applicability to real systems is the principal advantage of this method. Its principal disadvantage is the difficulty in solving certain nonlinear algebraic equations for the feedback gain matrix. The Suboptimal Output Feedback Method is a modification of the Optimal Output Feedback Method which admits direct noniterative solution for the feedback gain matrix; this feature constitutes the principal advantage of the method. Its principal disadvantage is that closed-loop stability, even for the design model, is not assured. Extensions to this method which make it potentially applicable to a wider class of systems, including large space structures, were reported [1; Sec. 6.2.3]. The Stochastic Optimal Output Feedback Method, currently under development, attempts to harmonize all available knowledge relevant to optimal output feedback. One unique feature of the method is its approach to the problem of spill-Unlike most other methods, which seek to weaken the coupling between critical and residual modes, this method enforces such coupling with the goal of having the residual modes "inherit" the stability properties designed into the critical modes.

Each of the design methods produced a stable controller for the comparison example. The time-domain performance (e.g., peak amplitudes, settling time) was best with the Suboptimal Output Feedback Method—as extended—and with the Stochastic Optimal Output Feedback Method, while frequency-domain performance (e.g., attenuation, distrubance rejection) was best with the Pole Assignment Method. The other methods indicated a relative lack of performance related to an inability to influence residual mode behavior [1; Chap. 8]. No attempt at a definitive scientific judgment on the relative merits of the five methods was made due to lack of an adequate data base for comparison. However, the results for the Suboptimal Output Feedback Method and the Stochastic Optimal Output Feedback Method were sufficiently encouraging to warrant recommendations for further active research on these two methods.

3.2 Scope of Current Research

Since the previous report [1] was written, additional research relevant to output feedback has focused on the Kosut method of suboptimal output feedback.[6] Extensions to the published Kosut methods that were reported in Reference 1 arose principally from the observation that the linear equation for the feedback gain matrix is algebraically consistent in all circumstances [1; Theorem 6-6]; i.e., it has an exact solution even when the coefficient matrix is singular. Since the coefficient matrix in the gain equation is a function

of the sensor observation matrix [1; Eq. (6-18)], this allows the method—as extended—to be applied to systems with arbitrary sensor configurations, in particular, those with observation matrices of less than maximum rank. Such sensor configurations can be expected when working with reduced-order models representing large space structures [1; Sec. 6.2.4].

The research reported in this section was directed toward developing an algorithm, suitable for use on a high-speed computer, for implementing the Kosut design method with the extensions reported in Reference 1. Implementation of the method requires:

- (1) Determining an "optimal" controller for a reference system having the same dynamics as the system to be controlled, but without explicit structure constraints on the controller [1; Sec. 6.1.2.1].
- (2) Solving the Kosut necessary conditions for suboptimality relative to the reference system [1; Secs. 6.1.2.2, 6.1.2.3].

Controller design for the reference system can be done using optimization criteria and design methods at the discretion of the designer. For example, a linear quadratic regulator design, for which reliable computational routines exist, is acceptable. Most such routines are based on some form of Potter's Method[11] for solving the algebraic Riccati equation for the feedback gain matrix. This part of the implementation is considered solved, and is not discussed further. Necessary conditions for suboptimality with the Kosut design consist of two significant linear algebraic equations. The first has the form [1; Sec. 6.1.2.2]

$$HP + PH^{T} + I = 0 (3-1)$$

where H:n×n is the (known) closed-loop system matrix of the optimal reference system, and P:n×n is a Lagrange multiplier matrix to be determined. It is assumed that the reference system design is stable; hence H is nonsingular. Under such hypotheses, reliable computational methods for solving equations of the form (3-1) exist. (For example, even Riccati-equation-solving algorithms, if used carefully, will work.) This part of the implementation is also considered solved, and is not discussed further.

The second of the Kosut necessary conditions, the gain matrix equation, gives difficulty. It has the form [1; Eq. (6-17)]

$$GA = B ag{3-2}$$

where the gain matrix $G:m\times \ell$ is to be determined. The matrices A and B are products, known a priori, with the special structure

$$A = CPC^{T} , B = F*PC^{T}$$
 (3-3)

where $C:\ell \times n$ is the sensor matrix, $P:n \times n$ is the (positive definite) solution of Eq. (3-1), and $F^*:m \times n$ is the state-to-control feedback matrix of the reference system. As a result of the extensions reported in Reference 1, the sensor

matrix C is allowed to be rank deficient; hence, the matrix A in Eq. (3-2) can be singular. Although the theory of solution for rank-deficient linear systems is completely documented in any good textbook on linear algebra (e.g., [12]), the numerical solution of such systems is a very delicate matter.

The purpose of this section is to detail the development of an algorithm for the numerical solution of rank-deficient linear systems of the form (3-2). In Section 3.3, certain pertinent topics from analysis are outlined in order to enable a concise presentation of the algorithm; the principal topics are the notions of a generalized inverse for, and the singular value decomposition of, an arbitrary rectangular matrix. In Section 3.4, numerical analysis topics relevant to development of the algorithm are discussed; principally, reasons why "obvious" approaches are likely to fail are given. The proposed algorithm is given in Section 3.5; included also is a calculation with the two-mode example of [1; Sec. 2.5], comparing results using the algorithm to results obtained in Reference 1 for the solution of Eq. (3-2). Section 3.6 gives the status of current research on numerical implementation of the Kosut design method.

3.3 Preliminaries From Analysis

It is sufficient to discuss $\operatorname{rank-deficient}$ linear vector equations of the form

$$Ax = b (3-4)$$

where A: $\mu \times \nu$ is rank deficient, b: $\mu \times 1$ is known, and x: $\nu \times 1$ is sought, since after transposition, Eq. (3-2) consists of a partitioned matrix of such equations. Moreover, for the application considered, it is only necessary to consider the situation that Eq. (3-4) is algebraically consistent, i.e.

$$rank [A:b] = rank [A]$$
 (3-5)

or, equivalently, that b is in the "column space" (vector space spanned by the column vectors) of A.

An introductory discussion of several quite sophisticated mathematical tools is very helpful in formulating a precise treatment of Eq. (3-4). These tools are used with increasing frequency, but are not always well understood. Moreover, the relevant literature is quite dispersed. Therefore, a careful presentation of the principal ideas is given next so that the basis for the algorithm to follow can be firmly established.

3.3.1 The Generalized Inverse of a Matrix

Moore [8] was the first to formulate successfully an appropriate meaning for the "inverse" of an arbitrary rectangular matrix. An equivalent formulation was given by Penrose [9]. Only the case of real matrices is discussed.

Theorem 3-1 (Penrose). Let the real matrix $A: \mu \times \nu$ have rank ρ . Then there exists a unique matrix $G: \nu \times \mu$ satisfying the conditions

$$AGA = A (3-6)$$

$$GAG = G (3-7)$$

$$(AG)^{T} = AG (3-8)$$

$$(GA)^{T} = GA (3-9)$$

The matrix G of Theorem 3-1 is called the Moore-Penrose inverse of A, and is denoted by A^{\dagger} . The reader should note that the literature is full of a bewildering assortment of nomenclatures and notations for various types of "generalized inverses", some nonunique, which have been devised for various applications. A lengthy list of these has been compiled by Rao and Mitra [13; Sec. 1.6]. Thus, any use of such generalized inverse concepts should be accompanied by a precise statement of the definition and important properties. Many are generalizations of the Moore-Penrose inverse. The "pseudoinverse" used in the discussion of the Modal Decoupling Method [1; Secs. 3.1.2, 3.5.2] is in fact identical to the Moore-Penrose inverse (cf Theorem 3-2). It also has an application to equations of the form (3-4) which have no exact solution. In such cases, the Moore-Penrose inverse "solution" $x = A^{\dagger}b$ is the optimum least squares approximation of minimum norm [15; Chap. 3] (i.e., the unique vector that simultaneously minimizes ||b-Ax|| and ||x||, where $||\cdot|$ is the Euclidean norm on E^{ν}).

It is important to be able to characterize the Moore-Penrose inverse in more concrete terms. For matrices of maximum rank, this is easy.

Theorem 3-2 (Greville [14]). Let matrices $B: \mu \times \rho$ and $C: \rho \times \nu$ have rank ρ . Then:

$$B^{\dagger} = (B^{T}B)^{-1}B^{T} \qquad (\rho \times \mu)$$
 (3-10)

and

$$c^{\dagger} = c^{T}(cc^{T})^{-1} \qquad (\vee \times \rho)$$
 (3-11)

Note that B^{\dagger} is a <u>left</u> inverse (the only one with rows in the row space of B^{T}) and that C^{\dagger} is a <u>right</u> inverse (the only one with columns in the column space of C^{T}). It is a fact [16] that every rectangular matrix can be decomposed as the product of two maximum rank matrices B, C of the type given in Theorem 3-2. This enables an easy description of the Moore-Penrose inverse for an arbitrary rectangular matrix.

Theorem 3-3 (Greville [16]). Let matrix A: $\mu \times \nu$ have rank ρ . Let B: $\mu \times \rho$ and C: $\rho \times \nu$ be matrices of rank ρ for which A = BC. Then

$$A^{\dagger} = C^{\dagger}B^{\dagger} \qquad (\nu \times \mu) \qquad (3-12)$$

where C^{\dagger} and B^{\dagger} represent the expressions in Eq. (3-11) and (3-10), respectively.

Note that the value of the product $C^{\dagger}B^{\dagger}$ is independent of the (nonunique) choice of matrices B, C in the decomposition A = BC.

The fundamental importance of the Moore-Penrose inverse for the study of algebraically consistent rank-deficient linear systems is evident from the following result.

Theorem 3-4 (Greville $^{[14]}$). Let matrices A: $\mu \times \nu$ and b: $\mu \times 1$ be given. Assume b is in the column space of A. Then, the general solution to Eq. (3-4) has the form

$$x = A^{\dagger}b + x_0$$
 (3-13)

where x_0 is orthogonal to the row space of A.

Note that the number of free parameters in the vector \mathbf{x}_0 is equal to the rank deficiency of A. In particular, when A is square and nonsingular, Eq. (3-13) reduces to the familiar (unique) solution: $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$. Theorem 3-4 is one basic result needed for development of an algorithm for solving Eq. (3-2). There are many other properties of the Moore-Penrose inverse that are peripheral to the main line of development, but which are essential for a complete understanding of how to work (both analytically and numerically) with it. Some of these properties are given in Section 3.7.1.

3.3.2 Singular Values and Singular Value Decomposition

The concept of an eigenvalue has no meaning for nonsquare rectangular matrices since the range and domain spaces are of different dimension. However, the related notion of a singular value has meaning for all matrices and turns out to be significant for many applications. The definition rests upon the following properties of symmetric matrix products.

Fact 3-5. For any matrix $A: \mu \times \nu$:

- (1) $AA^T: \mu \times \mu$ and $A^TA: \nu \times \nu$ have the same rank as A.
- (2) AA^{T} and $A^{T}A$ have the same nonzero eigenvalues.
- (3) The product of size $\max\{\mu,\nu\}$ has $\sigma+\delta$ zero eigenvalues, where $\sigma \stackrel{\triangle}{=} \text{number of zero eigenvalues for the product of size } \min\{\mu,\nu\}$, and $\delta \stackrel{\triangle}{=} \max\{\mu,\nu\}$ $\min\{\mu,\nu\}$.

The proof of Fact 3-5 is sketched in Section 3.7.2. Note that both AA^T and A^TA are positive semidefinite, and hence have real and nonnegative eigenvalues. [12] The singular values of a rectangular matrix A are the nonnegative square roots of the eigenvalues of $p(A,A^T)$, where $p(A,A^T)$ denotes either AA^T or A^TA , whichever is smallest in size. Determination of the rank of the coefficient matrix is an essential part of the numerical solution for rank-deficient linear systems (3-4). The following result shows the importance of singular values for rank determination.

Theorem 3-6. The rank of a matrix is equal to the number of its nonzero singular values.

This central property is made more explicit by a factorization which displays the singular values (singular value decomposition).

Theorem 3-7. Let A: $\mu \times \nu$ be a real matrix with rank ρ . Then:

(1) There exist orthogonal matrices $U: \mu \times \mu$ and $W: \nu \times \nu$ such that

where $\sigma_1 \geq \ldots \geq \sigma_\rho > 0$ are the nonzero singular values of A; and

(2) the columns of U are the eigenvectors of AA^{T} , and the columns of W are the eigenvectors of $A^{T}A$.

This result is due to Golub and Kahan [18]. An elegant proof has recently been given by Wilkinson [19]. This decomposition forms the basis of a reliable numerical procedure for determing the rank of a rectangular matrix. Moreover, it leads directly to a useful expression for the Moore-Penrose inverse [18].

Theorem 3-8. Let A: $\mu \times \nu$ have the singular value decomposition of Theorem 3-7. Then

$$\mathbf{A}^{\dagger} = \mathbf{W} \begin{bmatrix} \sigma_{1}^{-1} & 0 & | & & \\ & \ddots & & | & 0 \\ & & & \sigma_{1}^{-1} & | & \\ 0 & & & & \sigma_{2}^{-1} & | & \\ - & - & - & - & - & - & \\ 0 & & & & & 0 \end{bmatrix} \mathbf{U}^{\mathbf{T}}$$

It is easy to verify that this expression for A^{T} satisfies the Penrose conditions (Theorem 3-1). The result suggests that the nonzero singular values of A^{T} are the reciprocals of those for A. This supposition is correct, and its direct proof (Section 3.7.2) provides a good exercise in applying the properties of the Moore-Penrose inverse given in Section 3.7.1. All of the necessary tools for generating an algorithm for the numerical solution of rank-deficient linear systems of form (3-4) are now in hand.

It should be observed that singular values are significant for other purposes which, although outside the scope of this section, are relevant to the control of large space structures. Doyle [20] has shown that singular values, rather than eigenvalues, are the appropriate parameters to use in constructing measures of robustness relative to stability for multivariable systems.

3.4 Preliminaries from Numerical Analysis

Numerical solution of the gain matrix equation (3-2) involves three distinct conceptual processes.

- (1) Determination of the rank of the coefficient matrix A.
- (2) Finding a particular solution.
- (3) Finding the general solution incorporating arbitrary parameters corresponding to the rank deficiency of A.

The significance of the last process (3) should be emphasized. Results of previous work [1; Chaps. 6,8] proved that Eq. (3-2) is always solvable, and gave an example demonstrating how free parameters in the general solution associated with rank-deficient sensor matrices could be used to improve closed-loop system performance. Process (3) is needed to be able to realize this potential for performance improvement via computer implementation with high-order systems.

In theory, determination of matrix rank is easy. One simply reduces the matrix to row-echelon form by elementary row operations; the rank is the number of nonzero rows in the row-echelon matrix [12; Sec. 3.4]. The numerical determination of matrix rank, however, is a "notoriously dangerous" problem [21]. Gaussian elimination, the standard numerical technique for reducing a matrix of maximum rank to row-echelon form, breaks down when applied to rank-deficient matrices [22, Chap. 4; 23, Chap. 4; 24, p. 135]. Moreover, many rank determination algorithms based on alternative procedures are very unreliable due to rounding errors [24; p. 127]. To date the only method considered reliable for numerical determination of rank [21] is the Golub-Reinsch algorithm for computing the singular value decomposition [25]. A FORTRAN-IV implementation of this algorithm (SVD) has been thoroughly tested, and is included in the Eigensystem Package (EISPACK) produced by Argonne National Laboratory [26,27]. Theorems 3-6 and 3-7 provide the conceptual basis for rank determination by this method. It is worth observing that the technical details of development of the Golub-Reinsch algorithm[18,21] are far

more involved than those required simply to prove the existence of the singular value decomposition [19]; moreover, the two approaches taken are quite different.

Numerical determination of a particular solution to Eq. (3-2) is readily done using the singular value decomposition routine. In determining the rank of A, this routine computes the matrices U and W and nonzero singular values $\sigma_1, \ldots, \sigma_\rho$ (cf Theorem 3-7). It is then a simple calculation to obtain the Moore-Penrose inverse A^+ (cf Theorem 3-8). A particular solution A^+b to Eq. (3-4) follows (cf Theorem 3-4). By transposition and construction of a partitioned matrix, a particular solution

$$X_{p} = BA^{\dagger}$$
 (3-14)

to Eq. (3-2) is obtained. Certain numerical aspects of working with singular values should be observed here. Note that the procedure indicated in Theorem 3-8 for computing A^{\dagger} involves taking reciprocals of the singular values. Thus, this calculation is sensitive to the zero threshold parameter τ used by the SVD routine to determine which singular values are to be considered nonzero (i.e., if $0<\sigma_1<\tau$, then σ_1 is set to zero). This sensitivity property is not serious because the singular values, being defined in terms of eigenvalues of symmetric matrices, are relatively insensitive to small changes in those matrices $^{[21]}$. However, the actual computation of the singular values in the Golub-Reinsch algorithm explicitly eschews forming the products AA^T or A^TA to avoid squaring and possible elimination by roundoff of small singular values $^{[19,21,25]}$.

The general solution to Eq. (3-2) is obtained by adding to the particular solution X of Eq. (3-14) the general solution to the homogeneous equation

$$XA = 0 \tag{3-15}$$

This may be obtained by using a structure theorem similar to Theorem 6-3 of [1], but which is formulated in terms of matrix parameters in the singular value decomposition.

Theorem 3-9. Let $A: \ell \times n$, $X^0: m \times \ell$ be matrices. Denote $r \stackrel{\triangle}{=} rank$ (A). Then X^0 satisfies the homogeneous equation XA = 0 if and only if X^0 is a product of the form ΓU^T , where $\Gamma \equiv [0:\Gamma']$ is an $m \times \ell$ matrix whose first r columns are zero, and whose last ℓ -r columns are arbitrary, and U is an orthogonal $\ell \times \ell$ matrix whose columns are eigenvectors of AA^T .

The proof of this result parallels that given for Theorem 6-3 of [1]. To implement this result for the present application, one simply forms

$$X_{c} = \Gamma U^{T}$$
 (3-16)

with Γ as in Theorem 3-9 and U taken from the singular value decomposition of A (cf Theorem 3-7). The general solution to Eq. (3-2) is thus

$$X \equiv X_D + X_C = BA^{\dagger} + \Gamma U^{T}$$
 (3-17)

and contains $m \cdot (l-r)$ arbitrary parameters, where m is the number of control variables, l is the number of output variables, and r is the rank of the sensor matrix.

3.5 Algorithm for Solving Kosut's Gain Equation

3.5.1 Statement of Algorithm

A concise statement of an algorithm for solving the gain equation (3-2) associated with Kosut's design method as extended [1] can now be given. Recall that the sensor matrix $C:\ell \times n$ is a parameter of the reduced-order design model, while the feedback matrix $F*:m\times n$ and the multiplier matrix $P:n\times n$ are results from antecedent design calculations.

Step 1. Compute the rank of the coefficient matrix.

- (a) Compute $A \stackrel{\triangle}{=} CPC^{T}$. [cf Eq. (3-3)]
- (b) Compute the singular value decomposition of A, obtaining matrices U, W, and numbers σ_1 , ..., σ_r . [cf Theorem 3-7]
- (c) The (numerical) rank of A is r. [cf Theorem 3-6]

Step 2. Compute the rank of the augmented matrix.

- (a) Compute B $\stackrel{\triangle}{=}$ F*PC. [cf Eq. (3-3)]
- (b) Repeat Step 1 for the augmented matrix $\left[\frac{A}{B}\right]$
- (c) Check that rank $\left[\frac{A}{B}\right]$ is r. [1; Theorem 6-6]

If this test fails, terminate execution; an error has occurred.

Step 3. Find a particular solution.

- (a) Compute A^{\dagger}, using the parameters U, W, and σ_1 , ..., σ_r from Step 1(a). [cf Theorem 3-8]
- (b) Compute the solution $X_D = BA^{\dagger}$. [cf Eq. (3-14)]

Step 4. Find the general solution.

- (a) Select an $m \times (\ell-r)$ free-parameter matrix Γ' .
- (b) Form the m×r matrix $\Gamma \stackrel{\Delta}{=} [0|\Gamma']$.
- (c) Compute the homogeneous solution $X = \Gamma U^T$, using the matrix U from Step 1(a). [cf Theorem 3-9]
- (d) Compute the general solution $X = X_p + X_c$, using the results of Steps 3(b) and 4(c). [cf Eq. (3-17)]

General selection criteria for choosing the free-parameter matrix Γ' in Step 4(a) have yet to be established; specific examples have been given previously [1; Sec. 6.3.2].

3.5.2 Numerical Example

The general solution to the Kosut gain equation (3-2) has been determined analytically for the two-mass oscillator [1; Eq. (6-45)]:

$$G(\varepsilon,\delta) = \begin{bmatrix} -\sigma + \varepsilon & \frac{\psi_2}{\psi_1} & -\varepsilon \\ & & \\ -\delta & -\sigma + \delta & \frac{\psi_1}{\psi_2} \end{bmatrix}$$
(3-18)

where σ = 0.59708155, ψ_2/ψ_1 = -0.42539053, ψ_1/ψ_2 = -2.3507811.

This section compares result (3-18) with the general solution obtained by using the algorithm proposed previously in Section 3.5.1.

In applying the algorithm to this example, the matrices C, P, and F* computed in [1; Sec. 6.3] are used:

$$C = \begin{bmatrix} 0 & -0.85689010 \\ 0 & 0.36451293 \end{bmatrix}$$

$$P = \begin{bmatrix} 1.1373892 & -0.5 \\ -0.5 & 7.4375655 \end{bmatrix}$$

$$F* = \begin{bmatrix} 0.066389774 & 0.51609641 \\ -0.028241581 & -0.21954252 \end{bmatrix}$$

Results for each step of the algorithm are given in the following. For this comparison, calculations for the singular value decomposition were done using direct spectral analysis of the operators AA^T and A^TA , instead of using the more accurate Golub-Reinsch algorithm.

Step 1.

(a)
$$A = \begin{bmatrix} 5.4611116 & -2.3231052 \\ -2.3231052 & 0.98822692 \end{bmatrix}$$

(b) Eigenvalues of AA^T are
$$\lambda_1 = 41.593968$$
, $\lambda_2 = -7.0 \times 10^{-8}$ (λ_2 is "reset" to zero).

Singular values of A: $\sigma_1 = \lambda_1^{1/2} = 6.4493386$, $\sigma_2 \stackrel{\triangle}{=} 0$.

$$U = \begin{bmatrix} 0.92020151 & 0.39144501 \\ -0.39144499 & 0.92020150 \end{bmatrix} = W \stackrel{\triangle}{=} \begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}$$

(c) Numerical rank of A = 1.

Step 2.

(a) B =
$$\begin{bmatrix} -3.2607290 & 1.3870832 \\ 1.3870832 & -0.59005206 \end{bmatrix} \stackrel{\triangle}{=} \begin{bmatrix} \frac{b_1}{b_2} \\ \frac{b_2}{b_2} \end{bmatrix}$$

(b) Denote
$$A_1 \stackrel{\triangle}{=} \begin{bmatrix} -\frac{A}{b_1} \end{bmatrix}$$
, $A_2 \stackrel{\triangle}{=} \begin{bmatrix} -\frac{A}{b_2} \end{bmatrix}$.

Eigenvalues of $A_1A_1^T$ are $\ell_1 = 54.150321$, $\ell_2, \ell_3 = -10^{-7} \pm 1.6.558 \times 10^{-4}$ (ℓ_2, ℓ_3) "reset" to zero). Singular values of A_1 : $s_1 = \ell_1^{1/2} = 7.3586902$, $s_2 \stackrel{\triangle}{=} 0 \stackrel{\triangle}{=} s_3$. Eigenvalues of $A_2A_2^T$ are $\ell_1 = 43.866129$, $\ell_2, \ell_3 = -5 \times 10^{-8}$ $\pm 1.2.943 \times 10^{-4}$ (ℓ_2, ℓ_3 "reset" to zero). Singular values of A_2 : $s_1 = \ell_1^{1/2} = 6.623151$, $s_2 \stackrel{\triangle}{=} 0 \stackrel{\triangle}{=} s_3$.

(c) Numerical rank
$$\left[-\frac{A}{B}-\right]$$
 = numerical rank (A_1) = numerical rank (A_2) = 1

Step 3.

(a)
$$A^{\dagger} = U \begin{bmatrix} \sigma_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^{T} = \begin{bmatrix} 0.13129576 & -0.055851971 \\ -0.055851971 & 0.023758898 \end{bmatrix}$$

(b)
$$x_p \stackrel{\triangle}{=} BA^+ = \begin{bmatrix} -0.50559122 & 0.21507371 \\ 0.21507371 & -0.091490317 \end{bmatrix} \stackrel{\triangle}{=} \begin{bmatrix} x_1 & x_2 \\ x_2 & x_3 \end{bmatrix}$$

The reader may verify that, using X_p , we have $X_pA = B$ to within seven significant digits.

Step 4.

(a) Let
$$\gamma_1$$
, γ_2 be arbitrary $[m \cdot (\ell - r) = 2 \cdot (2 - 1) = 2]$

(b)
$$\Gamma = \begin{bmatrix} 0 & \gamma_1 \\ 0 & \gamma_2 \end{bmatrix}$$

(c)
$$X_c = \begin{bmatrix} 0.39144501 & \gamma_1 & 0.92020150 & \gamma_1 \\ 0.39144501 & \gamma_2 & 0.92020150 & \gamma_2 \end{bmatrix}$$

Note that:

$$\mathbf{x_{c}A} = \begin{bmatrix} -5 \times 10^{-9} \, \gamma_{1} & -4.4 \times 10^{-8} \, \gamma_{1} \\ -5 \times 10^{-9} \, \gamma_{2} & -4.4 \times 10^{-8} \, \gamma_{2} \end{bmatrix}$$

(d)
$$X = X_p + X_c = \begin{bmatrix} x_1 + \gamma_1 u_2 & x_2 + \gamma_1 u_4 \\ x_2 + \gamma_2 u_2 & x_3 + \gamma_2 u_4 \end{bmatrix}$$
 (3-19)

where the x_i, u_i are as defined in Steps 3(b) and 1(b), respectively.

Changing parameter variables from (γ_1, γ_2) to (ϵ, δ) as follows:

$$\varepsilon \stackrel{\Delta}{=} -x_2 - \gamma_1 u_4$$
 , $\delta \stackrel{\Delta}{=} -x_2 - \gamma_2 u_2$

enables (3-19) to be rewritten as

$$X = \begin{bmatrix} -\ell + \epsilon \rho & -\epsilon \\ -\delta & -\mu + \delta \tau \end{bmatrix}$$
 (3-20)

where $\ell \stackrel{\Delta}{=} 0.59708154$, $\mu \stackrel{\Delta}{=} 0.59708151$, $\rho \stackrel{\Delta}{=} -0.42539054$, $\tau \stackrel{\Delta}{=} -2.3507810$, which gives excellent agreement with the analytical solution (3-18).

3.6 Status of Current Research

The specific goal of the current phase of research related to Kosut's method of Suboptimal Output Feedback is to develop a systematic numerical procedure that is capable of realizing the potential for closed-loop performance improvement embodied in the extensions reported in Reference 1. The algorithm proposed in Section 3.5.1 is a necessary step. Current plans call for immediate coding of the algorithm for testing on the two-mass oscillator example of [1; Sec. 6.3], and on a higher order example (e.g., tetrahedral model; cf Appendix A). Once the reliability of this algorithm (including any modifications subsequently found necessary) has been established, subsequent research will focus on developing systematic techniques for selecting the free parameters in the general solution (3-19) so as to improve closed-loop performance.

3.7 Appendices

3.7.1 Additional Properties of the Moore-Penrose Inverse

The principal characterizations of the Moore-Penrose inverse have been given in Section 3.3.1. However, in order to be able to use this tool effectively, its properties need to be examined in greater detail. The difficulty in using some recent textbooks on the subject (e.g., [13], [15]) is that they present so much detail that a casual reader is hard pressed to focus on the properties important for his application. The properties listed in the following give insight into the Moore-Penrose inverse sufficient for the applications discussed in this report.

Let A: $\mu \times \nu$ be a rectangular matrix.

Theorem 3-10. Elementary Algebraic Properties.

- (1) Interchangeable: $(A^T)^+ = (A^+)^T$
- (2) Reflexive: $(A^{\dagger})^{\dagger} = A$
- (3) Consistent: If $\mu = \nu$, and A is nonsingular, then $A^{+} = A^{-1}$.

The basic reduction properties of the Moore-Penrose inverse may be viewed geometrically as follows:

Theorem 3-11. Elementary Geometric Properties.

- (1) Projective:
 - (a) $AA^{\dagger}: \mu \times \mu$ is a projection of E^{μ} onto the column space of A; i.e., $x \in \{\text{column space of } A\} \iff AA^{\dagger}x = x$
 - (b) $A^{\dagger}A: v \times v$ is a projection of E^{v} onto the column space of A^{\dagger} ; i.e., $y \in \{\text{column space of } A^{\dagger}\} \iff A^{\dagger}Ay = y$
- (2) Rank Preserving: rank (AA^{\dagger}) = rank (A^{\dagger}) = rank (A) = rank $(A^{\dagger}A)$
- (3) Symmetric: $(AA^{\dagger})^T = AA^{\dagger}$; $(A^{\dagger}A)^T = A^{\dagger}A$

The symmetric operators AA^T , A^TA and projection operators AA^{\dagger} , $A^{\dagger}A$ have the following important properties:

Theorem 3-12. Composite Properties.

(1) Reduction: $A^{T}(AA^{\dagger}) = A^{T} = (A^{\dagger}A)A^{T}$

$$A[A^{T}(A^{T})^{\dagger}] = A = [(A^{T})^{\dagger}A^{T}]A$$

(2) Inversion of symmetric operators: $(AA^T)^+ = (A^+)^TA^+$

$$(A^TA)^{\dagger} = A^{\dagger}(A^{\dagger})^T$$

(3) Inversion of projection operators: $(AA^{\dagger})^{\dagger} = AA^{\dagger}$

$$(A^{\dagger}A)^{\dagger} = A^{\dagger}A$$

Theorem 3-3 and properties (2) and (3) of Theorem 3-12 give special cases where the property:

$$(XY)^{\dagger} = Y^{\dagger}X^{\dagger}$$

holds. Unfortunately, this property does not hold in general [17].

3.7.2 Additional Results Relating to Singular Values

The proof of Fact 3-5 about symmetric matrix products is briefly sketched.

Proof of Fact 3-5.

To be specific, assume $\mu \leq \nu$. Denote $\rho \stackrel{\Delta}{=} \operatorname{rank}$ (A).

- (1) We show only that rank $(AA^T) = \operatorname{rank} A$. Since Range $(AA^T) \subset \operatorname{Range} (A)$: rank $(AA^T) \leq \operatorname{rank} (A)$. By the determinantal criterion for rank [12; Sec. 4.5], there exist row and column indices $1 \leq i_1 < \ldots < i_{\rho} \leq \mu$, $1 \leq j_1 < \ldots < j_{\rho} \leq \nu$ such that the $\rho \times \rho$ submatrix $A[i_1, \ldots, i_{\rho}|j_1, \ldots, j_{\rho}]$, consisting of rows i_1, \ldots, i_{ρ} and columns j_1, \ldots, j_{ρ} of A, has nonzero determinant. Denote $E \triangleq (AA^T)[i_1, \ldots, i_{\rho}|i_1, \ldots, i_{\rho}]$, and observe that $E = A[i_1, \ldots, i_{\rho}|1, \ldots, \nu]A^T[1, \ldots, \nu|i_1, \ldots, i_{\rho}]$. Using the formula for determinants of products of rectangular matrices [12; Sec. 4.6], we have
- $\det E = \sum_{1 \leq \alpha_1 \leq \dots \leq \alpha_{\rho} \leq \nu} \det A[i_1, \dots, i_{\rho} | \alpha_1, \dots, \alpha_{\rho}] \det A^T[\alpha_1, \dots, \alpha_{\rho} | i_1, \dots, i_{\rho}]$ $\geq \det^2 A[i_1, \dots, i_{\rho} | j_1, \dots, j_{\rho}] > 0,$

so rank $(AA^T) \ge \rho$.

- (2) It is sufficient to show that a nonzero eigenvalue of AA^T is also an eigenvalue of A^TA . If $\lambda \neq 0$ is an eigenvalue of AA^T , there exists $x \neq 0$ such that $A(A^Tx) = \lambda x$. Thus, $y \triangleq A^Tx \neq 0$, and $(A^TA)y = A^T\lambda x = \lambda y$.
- (3) If AA^T has a zero eigenvalue, then det $(AA^T) = 0$. Using result (1), we have rank $(A^TA) = \text{rank } (AA^T) < \mu$, so det (A^TA) is also zero. Moreover, using result (2), A^TA can have no nonzero eigenvalues distinct from those of AA^T .

The connection between the singular values of a matrix A and those of A^\dagger is clarified by noting the following fact regarding the inversion of eigenvalues for symmetric matrix products.

Lemma 3-13.

Assume $\lambda \neq 0$. Then:

- (1) λ is an eigenvalue of $AA^T \iff \lambda^{-1}$ is an eigenvalue of $(A^{\dagger})^T A^{\dagger}$.
- (2) λ is an eigenvalue of $A^TA \iff \lambda^{-1}$ is an eigenvalue of $A^{\dagger}(A^{\dagger})^T$.

Proof of Lemma 3-13.

(1) (=> part) By hypothesis, there exists $x \neq 0$ such that $AA^Tx = \lambda x$. Using reduction property (1) of Theorem 3-12: $A^{\dagger}\lambda x = (A^{\dagger}AA^T)x = A^Tx$. Moreover, x = Ay, with $y = A^Tx/\lambda$. Using property (1) of Theorems 3-10 and 3-12: $(A^{\dagger})^TA^{\dagger}\lambda x = [(A^T)^{\dagger}A^TA]y = Ay = x$.

- (2) (\Longrightarrow part) Set $B \stackrel{\triangle}{=} A^T$ and use (1) (\Longrightarrow).
- (1)/(2) (\leftarrow part) Set B = A[†], $\mu = \lambda^{-1}$ and use (2)/(1)(\Longrightarrow); property (2) of Theorem 3-10 is required.

This lemma leads immediately to the expected result about singular values:

Theorem 3-14. Assume $\sigma \neq 0$. Then:

 σ is a singular value for A $\iff \sigma^{-1}$ is a singular value for A[†].

LIST OF REFERENCES

- 1. Lin, J.G., Y.H. Lin, D.R. Hegg, T.L. Johnson, and J.E. Keat, <u>Theory of Design Methods</u>, <u>Interim Technical Report</u>, <u>Activity Controlled Structures Theory</u>, <u>Volume 1</u>, The Charles Stark Draper Laboratory, Inc., Report R-1249, April 1979.
- Lin, J.G., D.R. Hegg, Y.H. Lin, and J.E. Keat, "Output Feedback Control of Large Space Structures: An Investigation of Four Design Methods", 2nd VPI&SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft, Blacksburg, Virginia, June 21-23, 1979.
- 3. Canavin, J.R., "The Control of Spacecraft Vibration Using Multivariable Output Feedback", Paper 78-1419, AAIA/AAS Astrodynamics Conference, Palo Alto, California, August 7-9, 1978.
- Davison, E.J. and S.H. Wang, "On Pole Assignment in Linear Multivariable Systems Using Output Feedback", <u>IEEE Trans. Automatic Control</u>, Vol. AC-20, No. 4, pp. 516-518, August 1975.
- 5. Levine, W.S. and M. Athans, "On the Determination of the Optimal Constant Output Feedback Gains for Linear Multivariable Systems", IEEE Trans. Automatic Control, Vol AC-15, pp. 44-48, February 1970.
- Kosut, R.L., "Suboptimal Control of Linear Time-Invariant Systems Subject to Control Structure Constraints", <u>IEEE Trans. Automatic Control</u>, Vol. AC-15, pp. 557-563, October 1970.
- 7. Johnson, T.L., "Minimum-Variance Fixed-Form Compensation of Linear Systems", Proc. 17th IEEE Conference on Decision and Control, San Diego, California, January 10-12, 1979.
- 8. Moore, E.H., "On the Reciprocal of the General Algebraic Matrix", Amer. Math. Soc. Bull., Vol. 26, pp. 394-395, June 1920.
- 9. Penrose, R., "A Generalized Inverse for Matrices", Proc. Cambridge Philos. Soc., Vol. 51, Pt. 3, pp. 406-413, July 1955.

List of References (Cont.)

- 10. Athans, M. and P.L. Falb, Optimal Control. New York: McGraw-Hill, 1966.
- 11. Potter, J.E., "Matrix Quadratic Solutions", J. SIAM Appl. Math., Vol. 14, No. 3, pp. 496-501, May 1966.
- 12. Thrall, R.M. and L. Tornheim, <u>Vector Spaces and Matrices</u>. New York: Wiley, 1957.
- 13. Rao, C.R. and S.K. Mitra, Generalized Inverse of Matrices and its Applications. New York: Wiley, 1971.
- 14. Greville, T.N.E., "The Pseudoinverse of a Rectangular or Singular Matrix and its Application to the Solution of Systems of Linear Equations", SIAM Review, Vol. 1, No. 1, pp. 38-43, January 1959.
- 15. Ben-Israel, A. and T.N.E. Greville, <u>Generalized Inverses: Theory and Applications</u>. New York: Wiley, 1974.
- 16. Greville, T.N.E., "Some Applications of the Pseudoinverse of a Matrix", SIAM Review, Vol. 2, No. 1, pp. 15-22, January 1960.
- 17. Greville, T.N.E., "Note on the Generalized Inverse of a Matrix Product", SIAM Review, Vol. 8, No. 4, pp. 518-521, October 1966.
- 18. Golub, G. and W. Kahan, "Calculating the Singular Values and Pseudo-Inverse of a Matrix", <u>J. SIAM Numer. Anal.</u>, Vol. 2, No. 2, pp. 205-224, 1965.
- 19. Wilkinson, J.H., "Some Recent Advances in Numerical Linear Algebra",

 The State of the Art in Numerical Analysis, D. Jacobs, Ed., New York:

 Academic Press, 1977, pp. 3-53.
- 20. Doyle, J.C., "Robustness of Multiloop Linear Feedback Systems" Proc. 17th

 IEEE Conference on Decision and Control, San Diego, California,

 January 10-12, 1979.
- 21. Golub, G.H. and J.H. Wilkinson, "Ill-Conditioned Eigensystems and the Computation of the Jordan Canonical Form", SIAM Review, Vol. 18, No. 4, pp. 578-619, October 1976.
- 22. Wilkinson, J.H. <u>The Algebraic Eigenvalue Problem</u>. Oxford, England: Clarendon Press, 1965.
- 23. Blum, E.K., <u>Numerical Analysis and Computation: Theory and Practice</u>. Reading, Mass.: Addison-Wesley, 1972.
- 24. IBM System/360 Scientific Subroutine Package Version III Programmers Manual, GH20-0205-4, Fifth edition, August 1970.

List of References (Cont.)

- 25. Golub, G.H. and C. Reinsch, "Singular Value Decomposition and Least Squares Solutions", Handbook for Automatic Computation, Vol. II: Linear Algebra, J.H. Wilkinson, and C. Reinsch, Eds., New York: Springer-Verlag, pp. 134-151, 1971.
- 26. Garbow, B.S., J.M. Boyle, J.J. Dongarra, and C.B. Moler, Matrix Eigensystem Routines-EISPACK Guide Extension. New York: Springer-Verlag, 1977.
- 27. Parlett, B., "Progress in Numerical Analysis", SIAM Review, Vol. 20, No. 3, pp. 443-456, July 1978.

SECTION 4

STRUCTURAL DAMPING AUGMENTATION AND MODERN MODAL CONTROLLER DESIGN METHODOLOGY

4.1 Introduction

This section presents design methodology for active vibration suppression in a large space structure (LSS).[1,2]* The fundamental concepts are based on the Canavin[3,4] member-damper approach and the Balas[5-8] modern modal controller (MMC). Member dampers[9] are employed throughout the structure as vibration control devices. These devices may be thought of as electronic dashpots which can deliver a restoring force proportional to velocity. Structural damping augmentation (SDA) is achieved by configuring the member dampers such that the inherent damping of the structure is increased. This SDA is sufficient to offset control and observation spillover that result in MMC designs when composite closed-loop systems are evaluated. The tetrahedral model is employed as an example throughout this section.

4.2 Tetrahedral Model

The finite-element representation of the tetrahedral model (see Appendix A for details) is shown in Figure 4-1. This model contains 10 nodes,

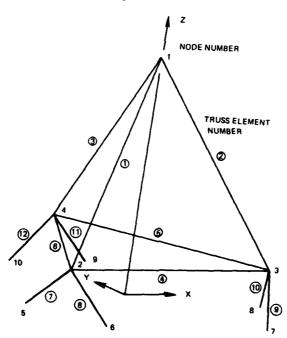


Figure 4-1. Finite-element representation of tetrahedral model.

Bracketed numerals refer to similarly numbered items in the List of References at the end of the section.

each with three degrees of freedom, and 12 truss elements which are capable of resisting only axial force. Masses are lumped at nodes 1 through 4. The node coordinates and element connectivities are listed in Appendix A. This structure is supported by pinned supports at nodes 5 through 10. An eigenvalue analysis of the nominal model yielded the results listed in Table 4-1.

Table 4-1. Modal natural frequencies nominal case.

Mode	rad/s	Hz
1	1.342	0.2136
2	1.665	0.2650
3	2.891	0.4601
4	2.957	0.4707
5	3.398	0.5408
6	4.204	0.6692
7	4.662	0.7420
8	4.755	0.7568
9	8.539	1.359
10	9.250	1.472
11	10.285	1.637
12	12.905	2.054

Six colocated sensor/actuator pairs are assumed to act in parallel with truss elements 7 through 12, as shown in Figure 4-2. The sensor is capable of providing relative velocity and position information. The actuator can exert a force in the axial direction of the truss element. For the purposes of this section, the sensor/actuator pairs are mechanized as member-damper devices. Each member damper senses the rate of change in length (velocity) of its corresponding truss element or strut. These devices also have the capability to exert equal and opposite forces at the extremes of each strut in opposition to the sensed velocity.

4.3 Structural Damping Augmentation

Henderson [3] investigated the capability of member dampers to suppress modal vibrations of a space frame structure. It was shown that the amount of damping per individual mode is unpredictable, as well as the fact that certain modes experience a limit in achievable modal damping. Canavin[4] showed that the closed-loop composite system with local member dampers is always

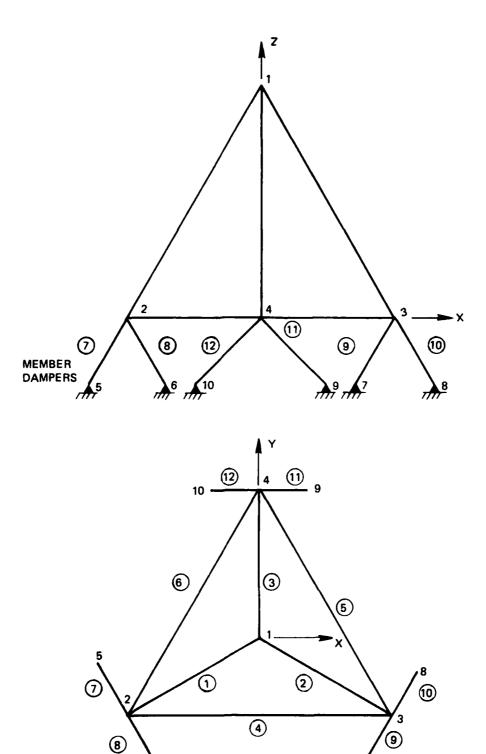


Figure 4-2. Member-damper locations on tetrahedral model.

Liapunov stable; hence, this type of velocity feedback cannot destabilize the structure even when the frequencies and mode shapes change drastically. Figure 4-3 presents a block diagram of an SDA controller with the tetrahedral model.

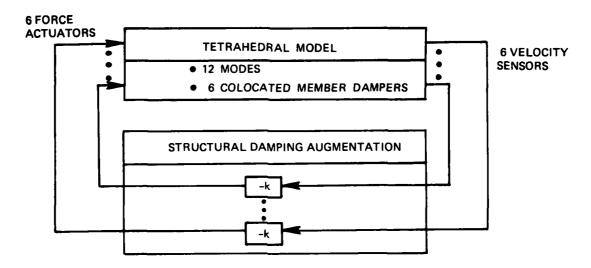


Figure 4-3. Structural damping augmentation.

The SDA controller multiplies each velocity output by a gain "-k" and provides the resulting command to the corresponding colocated force actuator. Table 4-2 shows the percent of critical damping achievable for various gains. Note that increasing the gain does not guarantee a corresponding increase in damping. Modes 1 and 2 initially increase in percent damping; however, for k > 10, the amount of damping decreases as the gain is continually increased. These two modes never reach 10 percent of critical damping. This result corresponds to poor observability or centrol authority with respect to sensor or actuator placement (see Section 2).

4.4 Modern Modal Controller

The modern modal controller design philosophy as developed by Balas [5-8] is represented by Figure 4-4. The procedures for determining the regulator and observer gains with a specified degree of stability are given in References 1 and 2. The resulting MMC is then analyzed with respect to a larger dimensional evaluation model, where it is possible that the observation and control spillover will cause an unstable closed-loop system. Investigations [1,2] have shown that evaluations with reduced-order models tend to be overly optimistic and, in general, MMC designs were unstable when evaluated with very high order models. This result promoted the concept of offsetting observation/control spillover with structural damping augmentation.

* Table 4-2. Percent critical damping, nominal case.

	Mode	1	2	3	7	5	9	7	8	6	10	11	12
Gain		(1.34 rad/s)	(1.67 rad/s)	(2.89 rad/s)	(2.96 rad/s) %	(3.40 rad/s)	(4.20 rad/s)	(4.66 rad/s)	(4.76 rad/s) %	(8.54 rad/s)	10 70	(10.29 rad/s)	(12.91 rad/s) %
	0.01	0.01 0.005	0.01	0.03	0.03	0.04	90.0	0.05	0.05	0.02	0.02	0.009	0.001
	0.1	0.05	0.1	0.28	0.34	0.44	0.59	0.52	0.48	0.20	0.16	0.088	0.010
	0.5	0.26	0.53	1.41	1.71	2.21	2.79	2.58	2.39	0.99	0.82	0.442	0.053
	1.0	0.52	1.05	2.83	3.42	4.42	5.95	5.17	4.79	1.97	1.63	0.883	0.106
	10.0 4.80	4.80	9.91	27.58	35.25	45.65	59.46	51.19	52.89	20.04	16.09	8.31	0.788
	100.0	3.13	3.36	100	100	100	100	100	100	6.01	5.76	3.73	0.259
<u>-</u>	1000.0 0.32	0.32	0.37							0.59	0.57	0.37	0.026
ŏ ——	10000.0 0.03	0.03	0.04							0.059	0.057	0.037	0.0026
100	100000.0 0.003	0.003	0.004	-				-	-	0.0059		0.0057 0.0037	0.00026

* 6 colocated velocity sensors and force actuators.

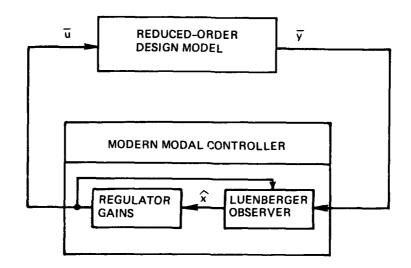


Figure 4-4. Modern modal controller design.

4.5 Structural Damping Augmentation and Modern Modal Controller

The adverse effect of observation and control spillover exhibited by MMC designs is always possible. If the structure had sufficient inherent damping, the effects of spillover could be eliminated. The closed-loop composite system poles would still migrate toward the right-hand s-plane, but not enough to destabilize it. In other words, the more inherent stability that a physical system has, the more energy required to destabilize it. The inherent structural damping can be enhanced by the SDA controller described previously.

For the tetrahedral model, it was arbitrarily decided that a MMC should provide at least 10 percent of critical damping in modes 1, 2, 4, and 5. Therefore, the reduced-order model consisted of only these four modes. Figure 4-5 shows the frequency spectrum of the four controlled modes and the remaining residual modes used in the 12-mode tetrahedral evaluation model.

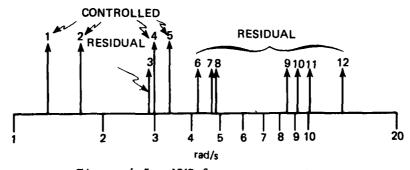


Figure 4-5. MMC frequency spectrum.

The four-mode design model was augmented with the SDA controller as shown in Figure 4-6 prior to designing the MMC. Therefore, the combined design plant

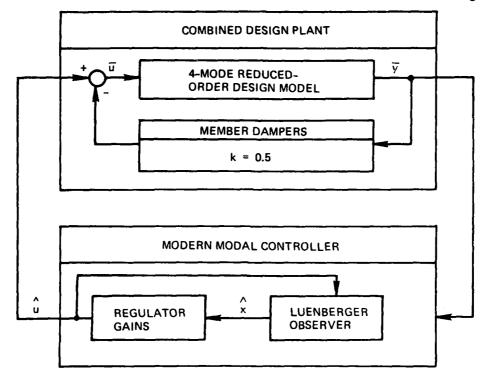


Figure 4-6. MMC design model with member dampers.

is representative of the physical system that the MMC must control. Using an alpha-shift of 1.75, and quadratic weights of $q_{ij}=1$, and $r_{ij}=10$, the regulator/observer gains were determined as described in References 1 and 2. The resulting regulator and observer poles are given in Table 4-3. This MMC utilizes the same six-member dampers shown in Figure 4-1. The evaluation is

Table 4-3. Four-mode MMC, regulator/observer poles.

	Regul	ator Poles		Observer Poles
Mode	A+BK	Damping (%)	Frequency (rad/s)	A-GC
1	-0.348 ± j1.34	25.1	1.39	-0.349 ± j1.34
2	-0.345 ± j1.66	20.3	1.70	-0.351 ± j1.67
4	-0.320 ± j2.96	10.8	2.97	-0.429 ± j2.96
5	-0.310 ± j3.40	9.1	3.41	-0.497 ± j3.41

performed by combining the SDA controller and the MMC with the full 12-mode model. Table 4-4 shows that the closed-loop eigenvalues of the combined SDA/MMC with the nominal 12-mode tetrahedral model are stable. The augmented structural damping did offset the adverse effect of observation and control spillover. However, the regulator- and observer-pole locations have changed as can be seen by comparing Tables 4-3 and 4-4. One complex pole pair only has 2.5 percent damping. It is not obvious that this is a regulator pole or an observer pole or, perhaps, the pole of residual mode 3. Therefore, an important consideration, which has been left out of this analysis, is the system performance achievable with this control-law design.

Table 4-4. SDA/MMC closed-loop eigenvalues for 12-mode tetrahedral model.

4.6 Summary and Conclusions

The local-member-damper concept (SDA controllers) is not satisfactory for providing high levels of modal damping; however, it is satisfactory when augmenting the structural damping for the purposes of eliminating the adverse effect of observation/control spillover. The MMC designs can yield optimistic results when analyzed on reduced-order models as compared to higher order evaluation models. The SDA/MMC design concept presented in this section provides a viable approach for designing stable closed-loop systems.

LIST OF REFERENCES

- 1. Strunce, R., and T. Henderson, Application of Design Methods, Interim
 Techincal Report, Actively Controlled Structures Theory, R-1249,
 Volume 2, The Charles Stark Draper Laboratory, Inc., April 1978.
- 2. Strunce, R.R., and T.C., Henderson, "Application of Modern Modal Controller Design to a Large Space Structure", Second VPI&SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft, Blacksburg, Virginia, June 21-23, 1979.
- 3. Henderson, T.C., and J.R. Canavin, "Damping Augmentations for Large Space Structures", Ninth AIAA/ASME Structures and Materials Conference, Paper No. 78-490, Bethesda, Maryland, April 1978.
- 4. Canavin, J.R., "Control Technology for Large Space Structures", Paper No. 78-1691 AIAA Conference on Large Space Platforms: Future Needs and Capabilities, Los Angeles, California, September 27-29, 1978.
- 5. Balas, M.J., "Feedback Control of Flexible Systems", <u>IEEE Trans. on</u> Automatic Control, Vol. AC-23, No. 4, pp. 673-679, August 1978.
- 6. Balas, M., "Modal Control of Certain Flexible Dynamic Systems," <u>2nd IFAC Symposium Distributed Farameter Control Systems</u>, Conventry, England, 1977.
- 7. Balas, M.J., Active Control of Flexible Systems, AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft, Blacksburg, VA., June 13-15, 1977.
- 8. Balas, M., and J. Canavin, "An Active Modal Control System Philosophy for a Class of Large Space Structures", AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft, Blacksburg, VA., June 13-15, 1977.
- 9. Passive and Active Suppression of Vibration Response in Precision
 Structures State-of-the-Art Assessment, R-1138, Vol. 2, The Charles
 Stark Draper Laboratory, Inc., Cambridge, Massachusetts, February 1978.

MISSION of Rome Air Development Center

RADC plans and executes research, development, test and selected acquisition programs in support of Command, Control Communications and Intelligence (C^3I) activities. Technical and engineering support within areas of technical competence is provided to ESD Program Offices (POs) and other ESD elements. The principal technical mission areas are communications, electromagnetic guidance and control, surveillance of ground and aerospace objects, intelligence data collection and handling, information system technology, ionospheric propagation, solid state sciences, microwave physics and electronic reliability, maintainability and compatibility.